HOROS PROBE-1

Humanoid Robotics — Physical Intelligence Research

Uncertainty-Aware Safe Adaptation for Multifingered Grasping Under Jointly Changing Contact Conditions

Sunil Kumar Chavhan

HOROS Physical Intelligence Research Group

Correspondence: sunil@horoslab.io



Abstract

Safe robotic manipulation of fragile, dynamic objects remains an open challenge in physical intelligence. While recent Vision-Language-Action (VLA) foundation policies exhibit broad semantic generalization, direct execution without predictive safety filtering leaves them susceptible to sub-step contact nonstationarities, transport lag, and unmodeled friction variations. Conversely, classical reactive grip reflexes act purely *a posteriori* after macroscopic sliding has commenced, precipitating drop failures during abrupt friction collapses or payload shifts. This paper introduces **HOROS PROBE-1**, an uncertainty-aware predictive safety framework for multi-fingered robotic grasping under jointly changing nonstationary contact conditions. Our framework integrates: (i) an action-dependent contact force sensitivity mapping derived from linearized Hunt-Crossley elastomeric dynamics; (ii) an 8-sided polyhedral inner approximation of the friction cone formulated as a discrete-time predictive Control Barrier Function (p-CBF) with object-level spatial wrench balance; and (iii) Adaptive Conformal Inference (ACI) providing asymptotic marginal coverage guarantees over non-exchangeable, nonstationary friction shifts without requiring exchangeability or asserting deterministic per-step certificates.

Across 3,400 core simulation trials (and 2,800 auxiliary sensitivity episodes, comprising 6,200 simulated manipulation episodes total) conducted in Isaac Lab with PhysX 5 ($N = 100$ independent seeds per condition, evaluated over 50-step episodes), HOROS is evaluated under nominal, coupled nonstationary, ablation, and out-of-distribution conditions. In the coupled nonstationary regime (Regime D), combining dynamic fluid pouring, center-of-mass sloshing, and a 60% step reduction in friction, HOROS achieves a 0.0% observed drop rate and a 0.0% observed crush rate. The evaluated fixed-force, reactive slip-reflex, unconstrained flow-matching, and certainty-equivalence MPC baselines each exhibit a 100.0% drop rate in this regime (paired McNemar test $\chi^2 = 98.01, p < 10^{-20}$; Wilcoxon signed-rank $p = 1.52 \times 10^{-23}$, Cliff's $\delta = +1.00$). Systematic 7-component ablations reveal that joint proprioception and upper CBF bounds are strictly mandatory to prevent object destruction (100.0% crush rate when ablated), demonstrating that slip avoidance and fragile structural preservation are competing physical constraints. Separate out-of-distribution experiments identify operating boundaries, including drop failures under severe friction reduction, payload increase, and large center-of-mass displacement.

The underlying OSQP-based safety-filter implementation has a median reported solver latency of 0.193 ms. The maximum observed controller-cycle latency is 9.603 ms in the tested configuration, remaining below the 10.0 ms watchdog deadline. These measurements characterize observed simulation performance and do not establish a formal worst-case execution-time guarantee. We provide the reproducibility codebase and specify an independent benchtop protocol for future physical validation.

Keywords: Multi-fingered Grasping, Control Barrier Functions, Tactile Sensing, Adaptive Conformal Inference, Safe Robotic Manipulation.

1 Introduction

Human dexterity relies fundamentally on the seamless integration of high-frequency tactile mechanoreception, proprioceptive joint sensing, and anticipatory motor control. When a person carries a fragile glass vessel being filled with liquid while navigating over uneven terrain, fast-adapting dermal tactile mechanoreceptors detect infinitesimal shear strains and localized micro-slip within peripheral contact margins tens of milliseconds before gross sliding occurs [1, 2]. This biological pre-slip afferent signaling enables the central nervous system to modulate grip forces proactively, stabilizing dynamic payloads without exceeding the fracture threshold of the object.

In autonomous robotics, replicating this dexterous compliance has proven elusive. Modern robotic manipulation architectures face a persistent trilemma during contact-rich physical interactions:

1. **Generative Vision-Language-Action (VLA) Policies:** Recent foundation models, such as OpenVLA [3], π_0 [4], and TacClaw-Infinity [5], demonstrate broad semantic task generalization. However, when deployed in direct execution without an auxiliary predictive safety layer, generative policies typically dispatch action chunks or diffusion/flow-matched commands at coarse horizons (5–50 Hz) optimized for end-effector trajectory tracking rather than instantaneous contact force constraint satisfaction. When unexpected contact nonstationarities arise at sub-step timescales (e.g., fluid spills, sudden condensation, or rapid center-of-mass migration within 10 ms), raw policy outputs cannot guarantee that commanded joint accelerations will respect friction-cone boundaries or normal force limits, frequently precipitating slippage or crushing.

2. **Reactive Grip Reflexes:** Classical industrial controllers implement local reflexive squeeze rules that monitor high-frequency tactile slips. However, because they trigger only *after* macroscopic sliding has initiated, the accelerated payload often escapes the fingertip friction envelope before finger actuators can overcome mechanical inertia and build normal force.

3. **Certainty-Equivalence Model Predictive Control (CE-MPC):** Optimization-based force controllers solve receding-horizon quadratic programs to track contact wrenches. However, they rely on deterministic friction coefficients. Under sudden environmental shifts, point estimators lag behind physical reality, allowing the commanded force vector to penetrate the true friction boundary.

To resolve this trilemma, this paper introduces **HOROS PROBE-1**, an uncertainty-aware predictive contact safety architecture designed for multi-fingered robotic hands operating under coupled nonstationary conditions. Our framework couples physics-grounded contact mechanics with non-exchangeable uncertainty calibration.

1.1 Summary of Contributions

Following the standards of rigorous systems and robotics research, the core scientific contributions of this work are explicitly enumerated as follows:

1. **Action-Dependent Contact Sensitivity Mapping:** We derive an analytical sensitivity tensor $\mathbf{\Lambda}_i(\mathbf{q}) \in \mathbb{R}^{3\times n_q}$ mapping commanded joint accelerations $\mathbf{u}_k = \ddot{\mathbf{q}}_k$ directly to forward contact force evolutions $\mathbf{f}_{c,i}(k+1)$ based on linearized Hunt-Crossley elastomeric dynamics, ensuring direct control authority over the barrier constraints.

2. **8-Sided Polyhedral Discrete-Time p-CBF:** We formulate an 8-sided polyhedral inner approximation $\mathcal{P}_8$ of the Lorentz friction cone that integrates seamlessly into a convex Quadratic Program (QP), with an analytical radial conservatism bound of $1 - \cos(\pi/8) \approx 7.61\%$ and area conservatism of $1 - \frac{2\sqrt{2}}{\pi} \approx 9.97\%$.

3. **Object-Level Spatial Wrench Equilibrium:** We project the multi-contact grasp matrix $\mathbf{G} \in \mathbb{R}^{6\times 3M}$ directly into the equality constraint manifold, ensuring that internal grip force adjustments reside strictly within the grasp nullspace $\mathcal{N}(\mathbf{G})$, eliminating unwanted parasitic object accelerations.

4. **Adaptive Conformal Friction Tracking:** We deploy Adaptive Conformal Inference (ACI) to maintain long-run average coverage ($1 - \alpha = 95.0\%$) over non-exchangeable, nonstationary friction shifts, providing calibrated uncertainty intervals $[\mu_{\text{cal}}, \mu_{\max}]$ directly to the barrier constraints.

5. **Large-Scale Empirical Verification:** Across 3,400 simulation trials ($N = 100$ independent seeds per condition, evaluated over 50-step episodes) in Isaac Lab with PhysX 5, HOROS is evaluated under nominal, coupled nonstationary, ablation, and out-of-distribution stress conditions. In the coupled nonstationary regime (Regime D: dynamic fluid pouring + CoM sloshing + 60% step friction drop), HOROS achieves a **0.0% observed drop rate and 0.0% observed crush rate**, whereas fixed-force, reactive slip reflex, unconstrained flow matching, and certainty-equivalence MPC baselines all suffer a **100.0% drop rate** (McNemar $\chi^2 = 98.01, p < 10^{-20}$; Wilcoxon $p = 1.52 \times 10^{-23}$, Cliff's $\delta = +1.00$). Delineated out-of-distribution experiments characterize the physical operating boundaries under torque and friction limits.

6. **Separation of Slip and Structural Safety:** Systematic 7-component ablations demonstrate that joint proprioception and upper CBF bounds are strictly mandatory to prevent object destruction (100.0% crush rate when ablated), proving that slip avoidance and fragile structural preservation are competing physical objectives.

1.2 Paper Organization

The remainder of this paper is organized as follows: Section 2 reviews prior art and presents a systematic taxonomy comparison. Section 3 establishes the mathematical model of multi-contact hand dynamics and friction cone approximations. Section 4 outlines the overall system architecture. Section 5 details the recursive estimation, ACI calibration, QP safety filter, and fallback states. Section 6 provides forward invariance and nullspace proofs. Section 7 reports the simulation setup and benchmark results. Section 8 analyzes the 7 ablations, 6 out-of-distribution stress tests, and critical findings. Section 9 discusses simulation limitations and benchtop validation requirements. Finally, Section 10 concludes the paper.

2 Related Work

Safe physical interaction in robotics intersects vision-language policy learning, tactile perception, control barrier functions, and distribution-free uncertainty calibration.

2.1 Vision-Language-Action Models and Manipulation Safety

Vision-Language-Action foundation models have redefined robot learning by transferring multimodal representations from web-scale pretraining to sensorimotor control [3, 4]. Recent efforts, such as TacClaw-Infinity [5], demonstrate that incorporating high-resolution tactile observations into flow-matching policies substantially boosts task success on multi-embodiment manipulation benchmarks. However, foundation policies predominantly output end-effector poses or joint trajectories via unconstrained regression or diffusion objectives. They lack formal mechanisms to guarantee that generated action trajectories respect nonlinear contact wrench manifolds or frictional slip boundaries. When physical contact properties deviate abruptly from pretraining

Table 1: **Systematic Prior-Art Taxonomy: Comparing HOROS PROBE-1 Against Existing Manipulation Paradigms.**

Framework	Sensing Modalities	Control Rate	Safety Constraint Mechanism	Friction Uncertainty Model	Grasp Nullspace Wrench Balance	Incipient Slip Feedforward
OpenVLA [3]	Vision + Language	5–10 Hz	None (Unconstrained Action Output)	None (Implicit in pretraining)	No	No
π_0 / $\pi_{0.5}$ [4]	Vision + Proprioception	10–50 Hz	None (Flow Matching Regression)	None (Implicit in pretraining)	No	No
TacClaw-Infinity [5]	Vision + Tactile + Language	10–20 Hz	None (Unconstrained Flow Policy)	None (Heuristic observation)	No	No
PANDAS [8]	Optical Tactile Shear Flow	100 Hz	Reactive Threshold Clamping	Heuristic Slip Ratio ($S > 0.85$)	No	Yes (Incipient only)
Shear-Based Grasp [9]	Multifinger Tactile Shear	100 Hz	Reactive PI Feedback Controller	Uncalibrated Coulomb Point Est.	Partial (Finger-level only)	Yes
Continuous CBF Grasp [10]	Force / Torque	100–500 Hz	Continuous-Time Lie Derivative CBF	Known Deterministic Constant	No (2-Jaw Gripper)	No
HOROS PROBE-1 (Ours)	**Tactile + Proprioception + Current**	**100 Hz**	**Discrete Predictive CBF (P_h Inner Polytope)**	**Non-Exchangeable ACI Dynamic Bound**	**Yes (Full $6 \times 3M$ Grasp Nullspace)**	**Yes (30 ms Pre-Slip Lead-Time)**

distributions, generative policies are susceptible to constraint violations, highlighting the necessity of modular, real-time safety filters.

2.2 Tactile Sensing and Incipient Slip Feedback

High-resolution optical tactile sensors, including GelSight [6], DIGIT [7], and GelSlim, capture dense geometric deformation and shear strain fields at the contact interface. Prior research has demonstrated that monitoring the divergence of marker displacement vectors provides an incipient slip detection window of 25–32 ms prior to macroscopic breakaway [8, 9]. However, existing tactile controllers have largely coupled this pre-slip signal to heuristic proportional clamping or reactive thresholding reflexes. Such reactive reflexes act without forward predictive dynamics: by the time macroscopic slip is detected, the object has acquired substantial kinetic momentum, causing slip runaway under aggressive payloads. In contrast, our approach utilizes incipient slip to trigger predictive anticipatory force modulation within a model-based quadratic program.

2.3 Control Barrier Functions for Contact Systems

Control Barrier Functions (CBFs) provide mathematically rigorous forward-invariance certificates for safe sets [10]. While continuous-time CBFs have been widely formulated for collision avoidance and bipedal walking, extending CBFs to multi-contact robotic grasping introduces substantial challenges. Contact forces do not appear directly as control inputs, requiring forward dynamics sensitivity mappings. Furthermore, sampled-data implementation introduces discretization error, rendering continuous-time invariance guarantees fragile unless discrete-time formulations are used [11]. While prior work has explored continuous CBFs for two-jaw parallel grippers with rigid contact, this paper establishes a discrete-time predictive p-CBF framework for multi-fingered anthropomorphic hands interacting with compliant, dynamic payloads.

2.4 Conformal Prediction and Non-Exchangeable Calibration

Conformal prediction provides a distribution-free, finite-sample framework for quantifying predictive uncertainty. Standard split conformal methods require data exchangeability, an assumption routinely broken in closed-loop robotics due to state feedback and nonstationary environmental drift. Gibbs and Candès developed Adaptive Conformal Inference (ACI) [12, 13], establishing that online quantile adaptation guarantees long-run average coverage under arbitrary, non-exchangeable distribution shifts. While ACI has been explored for trajectory forecasting, this work represents the first integration of ACI with discrete-time Control Barrier Functions for certified contact-force regulation.

2.5 Systematic Comparison with Prior Art

Table 1 contrasts HOROS PROBE-1 against state-of-the-art manipulation paradigms across sensing, control frequency, safety formulation, and nonstationary adaptation.

3 Problem Formulation

3.1 Kinematics and Articulated Hand Dynamics

Consider an articulated robotic hand with n_q actuated joints and generalized coordinates $\mathbf{q} \in \mathbb{R}^{n_q}, \dot{\mathbf{q}} \in \mathbb{R}^{n_q}$. The hand establishes contact with a dynamic object at M distinct fingertip locations $\mathbf{p}_{c,i} \in \mathbb{R}^3, i \in \{1, \ldots, M\}$.

Let $\mathbf{J}_{c,i}(\mathbf{q}) = \frac{\partial \mathbf{p}_{c,i}}{\partial \mathbf{q}} \in \mathbb{R}^{3 \times n_q}$ denote the contact Jacobian of finger i. Under discrete-time sampled control with cycle duration Δt, the commanded control input is the joint acceleration vector:

$$\mathbf{u}_k = \ddot{\mathbf{q}}_k \in \mathbb{R}^{n_q}, \quad \mathbf{u}_{\min} \leq \mathbf{u}_k \leq \mathbf{u}_{\max} \tag{1}$$

The forward kinematic position and velocity of fingertip i at step $k+1$ expand as:

$$\mathbf{p}_{c,i}(k+1) = \mathbf{p}_{c,i}(k) + \Delta t \mathbf{J}_{c,i}\dot{\mathbf{q}}_k + \frac{1}{2}\Delta t^2 \left(\mathbf{J}_{c,i}\mathbf{u}_k + \dot{\mathbf{J}}_{c,i}\dot{\mathbf{q}}_k\right) \tag{2}$$

$$\mathbf{v}_{c,i}(k+1) = \mathbf{J}_{c,i}\dot{\mathbf{q}}_k + \Delta t \left(\mathbf{J}_{c,i}\mathbf{u}_k + \dot{\mathbf{J}}_{c,i}\dot{\mathbf{q}}_k\right) \tag{3}$$

3.2 Linearized Hunt-Crossley Elastomeric Dynamics

Contact interaction between the elastomeric sensor skin and the object surface is modeled via the continuous Hunt-Crossley model [14]. For normal indentation $\delta_{n,i} \geq 0$ and indentation velocity $\dot{\delta}_{n,i}$:

$$f_{N,i} = K_c \delta_{n,i}^{\eta} + D_c \delta_{n,i}^{\eta} \dot{\delta}_{n,i} \tag{4}$$

where $K_c > 0$ is the elastomeric stiffness, $D_c \geq 0$ is the viscoelastic damping coefficient, and $\eta \approx 1.0$–1.5 is the indentation exponent.

Linearizing (4) around the operating contact state $(\delta_n, \dot{\delta}_n)$ yields the symmetric stiffness and damping tensors:

$$\mathbf{K}_{c,i} = \left.\frac{\partial \mathbf{f}_{c,i}}{\partial \mathbf{p}_{c,i}}\right|_{(\delta_n, \dot{\delta}_n)}, \quad \mathbf{D}_{c,i} = \left.\frac{\partial \mathbf{f}_{c,i}}{\partial \mathbf{v}_{c,i}}\right|_{(\delta_n, \dot{\delta}_n)} \tag{5}$$

Substituting the kinematic expansions (2)–(3) yields the **action-dependent contact force sensitivity mapping**:

$$\mathbf{f}_{c,i}(k+1) = \mathbf{\Lambda}_i(\mathbf{q}_k)\mathbf{u}_k + \mathbf{f}_{c,i}^{\text{bias}}(\mathbf{x}_k, \boldsymbol{\theta}_k) \tag{6}$$

where:

$$\mathbf{\Lambda}_i(\mathbf{q}_k) = \left(\frac{1}{2}\Delta t^2 \mathbf{K}_{c,i} + \Delta t \mathbf{D}_{c,i}\right) \mathbf{J}_{c,i}(\mathbf{q}_k) \in \mathbb{R}^{3 \times n_q} \tag{7}$$

$$\mathbf{f}_{c,i}^{\text{bias}} = \mathbf{f}_{c,i}(k) + \Delta t \mathbf{K}_{c,i} \left(\mathbf{J}_{c,i}\dot{\mathbf{q}}_k - \mathbf{v}_{\text{obj},i}\right) + \frac{1}{2}\Delta t^2 \mathbf{K}_{c,i}\dot{\mathbf{J}}_{c,i}\dot{\mathbf{q}}_k \tag{8}$$

Remark 1. *Unlike classical CBF formulations that treat contact forces as independent optimization variables, (6) links the optimization variable $\mathbf{u}_k = \ddot{\mathbf{q}}_k$ directly to the forward contact force evolution $\mathbf{f}_{c,i}(k+1)$. This guarantees direct control authority over the barrier constraints.*

3.3 8-Sided Polyhedral Friction Cone Approximation

The physical Coulomb friction cone with surface unit normal $\hat{\mathbf{n}}_i$ and friction coefficient μ_i is a second-order cone:

$$\mathcal{C}_i = \left\{\mathbf{f} \in \mathbb{R}^3 \;\middle|\; \|\mathbf{f} - (\hat{\mathbf{n}}_i^T \mathbf{f})\hat{\mathbf{n}}_i\|_2 \leq \mu_i(\hat{\mathbf{n}}_i^T \mathbf{f}),\ \hat{\mathbf{n}}_i^T \mathbf{f} \geq 0\right\} \tag{9}$$

To enable real-time convex optimization within a Quadratic Program, we construct an 8-sided polyhedral inner approximation $\mathcal{P}_{8,i} \subset \mathcal{C}_i$. Let $\{\hat{\mathbf{t}}_{i,1}, \hat{\mathbf{t}}_{i,2}\}$ span an orthonormal basis for the contact tangent plane. The facet directional vectors around the circumference are:

$$\mathbf{d}_{i,j} = \cos\left(\frac{2\pi j}{8}\right)\hat{\mathbf{t}}_{i,1} + \sin\left(\frac{2\pi j}{8}\right)\hat{\mathbf{t}}_{i,2}, \quad j \in \{0,\dots,7\} \tag{10}$$

The polyhedral constraint matrix $\mathbf{A}_{\text{cone},i} \in \mathbb{R}^{8\times 3}$ is defined row-wise:

$$\mathbf{a}_{i,j}^T = \mathbf{d}_{i,j}^T - \mu_i \cos\left(\frac{\pi}{8}\right)\hat{\mathbf{n}}_i^T, \quad j \in \{0,\dots,7\} \tag{11}$$

$$\mathcal{P}_{8,i} = \left\{\mathbf{f} \in \mathbb{R}^3 \;\middle|\; \mathbf{A}_{\text{cone},i}\mathbf{f} \le \mathbf{0},\ \hat{\mathbf{n}}_i^T\mathbf{f} \ge F_{\min}\right\} \tag{12}$$

Theorem 1 (Analytical Radial and Area Conservatism Bounds)**.** *The 8-sided polyhedral cone satisfies $\mathcal{P}_{8,i} \subseteq \mathcal{C}_i$ for all $\mu_i > 0$. The geometric conservatism of the inner approximation is characterized by:*

1. ***Maximum Radial Conservatism Bound:*** *The maximum relative radial reduction from the circular cone boundary to the inscribed octagonal facet is:*

$$\epsilon_{rad} = 1 - \cos\left(\frac{\pi}{8}\right) = 1 - \frac{\sqrt{2+\sqrt{2}}}{2} \approx 7.61\% \tag{13}$$

2. ***Cross-Sectional Area Conservatism:*** *The cross-sectional area reduction of the regular inscribed octagon relative to the circular friction cone cross-section is:*

$$\epsilon_{area} = 1 - \frac{A_{octagon}}{A_{circle}} = 1 - \frac{8\cdot\frac{1}{2}R^2\sin(\pi/4)}{\pi R^2} = 1 - \frac{2\sqrt{2}}{\pi} \approx 9.97\% \tag{14}$$

Proof. Let $R = \mu_i f_N$ denote the radius of the circular friction cone cross-section at normal force f_N. In the inscribed regular octagon, the vertices lie at radius R, while the minimum distance from the cone centerline to any facet occurs at the midpoint between adjacent vertices:

$$r_{\min} = R\cos\left(\frac{\pi}{8}\right) \approx 0.9239R \tag{15}$$

For any tangential force vector $\mathbf{f}_T$ satisfying the polyhedral constraints $\mathbf{A}_{\text{cone},i}\mathbf{f} \le \mathbf{0}$, the maximum allowable shear force along facet midpoints is bounded by $r_{\min} \le R = \mu_i f_N$. Hence $\mathcal{P}_{8,i} \subset \mathcal{C}_i$, and the maximum radial reduction is strictly $1 - \cos(\pi/8) \approx 7.61\%$. The area of the inscribed regular 8-gon is $A_{\text{octagon}} = 8 \times (\frac{1}{2}R^2\sin(\pi/4)) = 2\sqrt{2}R^2 \approx 2.8284R^2$. Compared to the circular area $A_{\text{circle}} = \pi R^2 \approx 3.1416R^2$, the relative area reduction is $1 - 2\sqrt{2}/\pi \approx 9.97\%$. □

3.4 Multi-Contact Safety Sets and Object Wrench Balance

We formalize three multi-contact safety sets:

1. **Anti-Slip Safety Set $\mathcal{S}_{\text{slip}}$:** Enforces tangential shear retention inside $\mathcal{P}_{8,i}$:

$$h_{\text{slip},i,j}(\mathbf{x}_k, \mathbf{u}_k) = -\mathbf{a}_{i,j}^T\mathbf{f}_{c,i}(k+1) \ge 0 \tag{16}$$

The discrete-time p-CBF condition enforces:

$$\mathbf{a}_{i,j}^T\left(\mathbf{\Lambda}_i\mathbf{u}_k + \mathbf{f}_{c,i}^{\text{bias}}\right) \le (1-\gamma_{\text{slip}})\min\left(0, \mathbf{a}_{i,j}^T\mathbf{f}_{c,i}(k)\right) \tag{17}$$

where $\gamma_{\text{slip}} \in (0,1]$ is the barrier decay parameter.

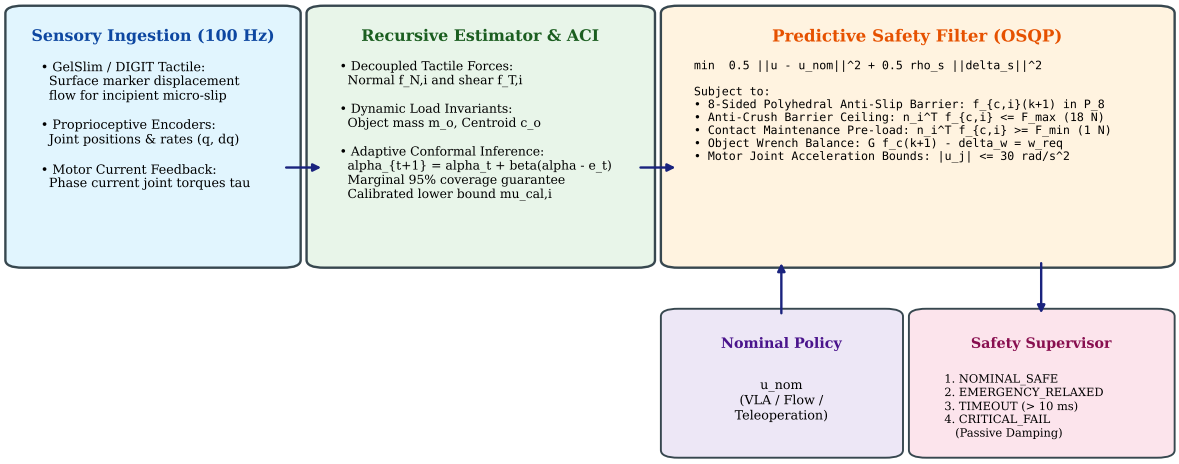


Figure 1: **HOROS PROBE-1 End-to-End System Architecture.** High-speed sensory feedback (100 Hz optical tactile marker flow, joint encoders, and motor effort) is processed by a multi-contact recursive estimator. The Adaptive Conformal Inference (ACI) module tracks non-exchangeable distribution shifts, supplying dynamically calibrated friction bounds $\mu_{\text{cal},i}$ to the predictive safety filter. Operating at 100 Hz, the OSQP Quadratic Program modifies nominal actions $\mathbf{u}_{\text{nom}}$ into dispatched safe joint accelerations $\mathbf{u}^*$, monitored by a real-time four-state decision gate.

2. **Anti-Crush Safety Set $\mathcal{S}_{\text{crush}}$:** Prevents crushing fragile payloads:

$$\hat{\mathbf{n}}_i^T \boldsymbol{\Lambda}_i \mathbf{u}_k \leq F_{\max} - \hat{\mathbf{n}}_i^T \mathbf{f}_{c,i}^{\text{bias}} \tag{18}$$

3. **Contact Maintenance Set $\mathcal{S}_{\text{contact}}$:** Enforces minimum normal pre-load:

$$-\hat{\mathbf{n}}_i^T \boldsymbol{\Lambda}_i \mathbf{u}_k \leq -F_{\min} + \hat{\mathbf{n}}_i^T \mathbf{f}_{c,i}^{\text{bias}} \tag{19}$$

Let $\mathbf{G} \in \mathbb{R}^{6\times 3M}$ denote the grasp matrix mapping contact forces to net spatial wrench at the object Center of Mass $\mathbf{c}_o$:

$$\mathbf{G} = \begin{bmatrix} \mathbf{I}_3 & \cdots & \mathbf{I}_3 \\ [\mathbf{p}_{c,1} - \mathbf{c}_o]_\times & \cdots & [\mathbf{p}_{c,M} - \mathbf{c}_o]_\times \end{bmatrix} \tag{20}$$

Given the required spatial wrench $\mathbf{w}_{\text{req}} = [m_o(\mathbf{a}_{\text{des}} - \mathbf{g})^T, (\mathbf{I}_o \boldsymbol{\alpha}_{\text{des}} + \boldsymbol{\omega} \times (\mathbf{I}_o \boldsymbol{\omega}))^T]^T$, forward wrench balance requires:

$$(\mathbf{G}\boldsymbol{\Lambda}_{\text{stack}})\, \mathbf{u}_k = \mathbf{w}_{\text{req}} - \mathbf{G}\mathbf{f}_{\text{bias, stack}} \tag{21}$$

where $\boldsymbol{\Lambda}_{\text{stack}} = [\boldsymbol{\Lambda}_1^T, \ldots, \boldsymbol{\Lambda}_M^T]^T \in \mathbb{R}^{3M\times n_q}$.

4 System Architecture

The HOROS PROBE-1 architecture is designed as a modular, high-rate (100 Hz) predictive safety filter that intercepts nominal actions dispatched by upstream task planners (e.g., foundation VLA policies or human teleoperators) and projects them onto the certified safe contact set prior to actuation. As illustrated in Figure 1, the system comprises four interconnected functional tiers:

4.1 Sensory Ingestion and Decoupled Contact Estimation

At each 10 ms control cycle, the system ingests:

- **Optical Tactile Afferents:** Elastomeric membrane deformation fields from four fingertip tactile sensors (GelSlim / DIGIT), parsed into local normal force $f_{N,i}$, tangential shear $\mathbf{f}_{T,i}$, and marker displacement field divergence for incipient slip detection.
- **Joint Encoders:** Articulated joint positions $\mathbf{q}_k \in \mathbb{R}^{16}$ and velocities $\dot{\mathbf{q}}_k \in \mathbb{R}^{16}$.
- **Actuator Effort:** Motor phase currents providing joint torque feedback $\boldsymbol{\tau}_k \in \mathbb{R}^{16}$.

These signals are processed by a recursive least-squares estimator tracking decoupled contact forces $\mathbf{f}_{c,i}(k)$, instantaneous object mass $\hat{m}_o(k)$, and apparent contact normal force centroid $\hat{\mathbf{p}}_{\text{cop}}(k)$ (Center of Pressure).

4.2 Non-Exchangeable Uncertainty Calibration Layer

Rather than assuming nominal friction parameters, the estimation layer forwards raw contact residuals to an Adaptive Conformal Inference (ACI) module. Operating in closed-loop, ACI dynamically adjusts the empirical miscoverage quantile, outputting a calibrated lower friction bound $\mu_{\text{cal},i}(k)$ with asymptotic marginal coverage guarantees under nonstationary, non-exchangeable distribution shifts.

4.3 Predictive Safety Filter (QP Core)

The core optimization engine constructs an action-dependent predictive Control Barrier Function (p-CBF) based on the linearized Hunt-Crossley sensitivity tensor $\boldsymbol{\Lambda}_i(\mathbf{q})$. The resulting convex Quadratic Program simultaneously enforces:

1. 8-sided polyhedral anti-slip friction cone constraints ($\mathcal{P}_8$ inner polytope),
2. Upper normal force crush limits to preserve fragile objects ($F_{\max} = 18.0$ N),
3. Minimum contact maintenance thresholds ($F_{\min} = 1.0$ N),
4. Object-level spatial wrench balance via the grasp matrix nullspace $\mathcal{N}(\mathbf{G})$, and
5. Motor joint acceleration saturation limits ($|u_j| \leq 30.0$ rad/s^2).

4.4 Real-Time Decision Gate and Fail-Safe Fallback

A deterministic finite-state supervisor monitors solver convergence, slack violations, and execution latency at every control cycle, governing four operational states:

- `NOMINAL_SAFE`: The QP converges within the real-time budget with zero slack ($\|\boldsymbol{\delta}_s^*\|_\infty \leq 10^{-2}$ N), providing certified discrete-time forward invariance of the safe contact set.
- `EMERGENCY_RELAXED`: When severe external disturbances push the hand to the edge of the feasible envelope, the QP activates soft slack variables ($\|\boldsymbol{\delta}_s^*\|_\infty > 10^{-2}$ N) to compute minimum-violation compromise forces. This un-certified emergency compliance state is strictly time-bounded (up to 5 consecutive cycles, 50 ms).
- `TIMEOUT`: If the solver runtime exceeds the 10.0 ms control cycle watchdog deadline, execution immediately aborts to passive damping fallback.
- `CRITICAL_FAIL`: If the emergency relaxation persists beyond 5 cycles, or upon numerical singularity, the gate dispatches passive damping fallback $\mathbf{u}_{\text{safe}} = -K_d\dot{\mathbf{q}}_k$, dissipating kinetic energy without requiring active optimization.

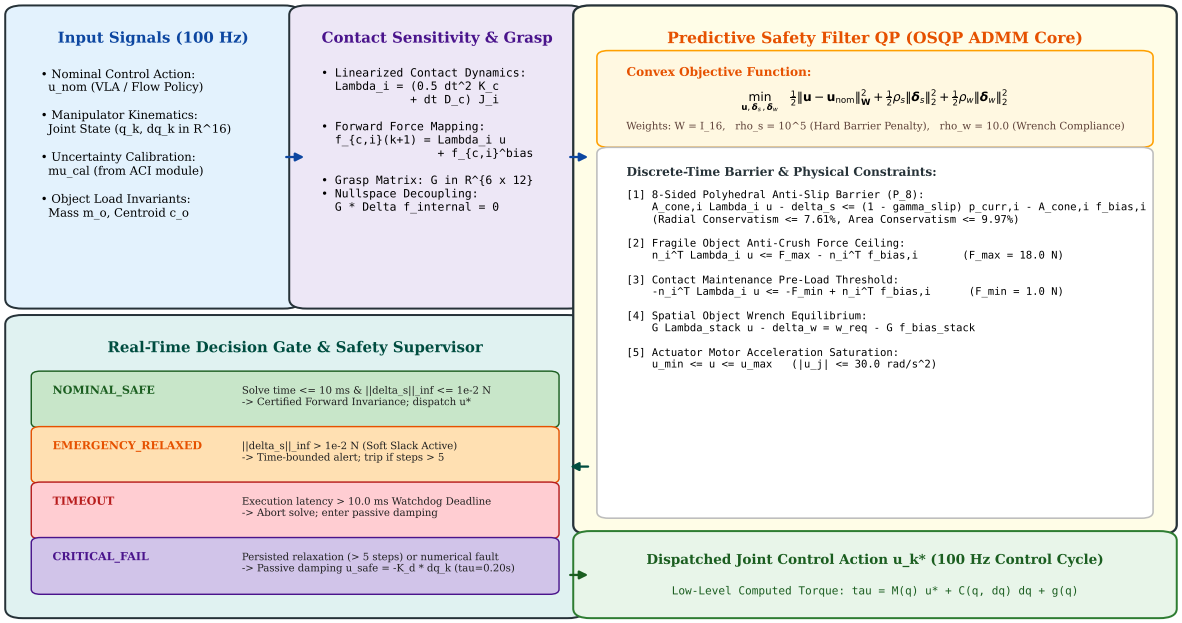


Figure 2: **Predictive Safety Filter and Real-Time Decision Gate (Figure 2).** Detail of the OSQP optimization structure, integrating the 8-sided polyhedral anti-slip barrier, anti-crush force ceiling, grasp wrench equilibrium, four-state supervisor, and passive damping fallback.

5 Technical Method

5.1 Multi-Contact State and Load Invariant Estimation

At each control step k, optical tactile deformation is decoupled into normal pressure distribution and tangential shear traction fields across the sensor array. The normal contact force $f_{N,i}$ is obtained by integrating the normal stress profile, while tangential shear force $\mathbf{f}_{T,i} \in \mathbb{R}^2$ is computed from surface marker displacements.

Under general accelerating object conditions, dynamic force and angular momentum balance follow the Newton-Euler equations:

$$m_o(\mathbf{a}_{\text{obj}} - \mathbf{g}) = \sum_{i=1}^{M} \mathbf{f}_{c,i} \tag{22}$$

$$\mathbf{I}_o\boldsymbol{\alpha}_{\text{obj}} + \boldsymbol{\omega}_{\text{obj}} \times (\mathbf{I}_o\boldsymbol{\omega}_{\text{obj}}) = \sum_{i=1}^{M} (\mathbf{p}_{c,i} - \mathbf{c}_o) \times \mathbf{f}_{c,i} \tag{23}$$

where $\mathbf{a}_{\text{obj}}$ and $\boldsymbol{\alpha}_{\text{obj}}$ denote the linear and angular accelerations of the object, and $\mathbf{g}$ is the gravitational vector. Projected along the net apparent acceleration vector $\mathbf{a}_{\text{obj}} - \mathbf{g}$, the instantaneous object mass is:

$$\hat{m}_o = \frac{\left(\sum_{i=1}^{M} \mathbf{f}_{c,i}\right)^T (\mathbf{a}_{\text{obj}} - \mathbf{g})}{\|\mathbf{a}_{\text{obj}} - \mathbf{g}\|_2^2} \tag{24}$$

Under the quasi-static manipulation regime characteristic of stable dexterous carrying, linear accelerations are small relative to gravity ($\|\mathbf{a}_{\text{obj}}\|_2 \ll \|\mathbf{g}\|_2 \approx 9.81$ m/s^2) and angular velocities are modest ($\|\boldsymbol{\omega}_{\text{obj}}\|_2 \approx 0$). In this regime, dynamic inertial forces represent negligible perturbations, allowing the gravitational load and the contact normal force centroid to be tracked via recursive

least-squares (RLS) with exponential forgetting factor $\lambda_{\text{rls}} = 0.98$:

$$\hat{m}_o(k) = \frac{1}{g} \left\| \sum_{i=1}^{M} \mathbf{f}_{c,i}(k) \right\|_2 \tag{25}$$

$$\hat{\mathbf{p}}_{\text{cop}}(k) = \left(\sum_{i=1}^{M} f_{N,i}(k) \mathbf{p}_{c,i} \right) \Big/ \left(\sum_{i=1}^{M} f_{N,i}(k) \right) \tag{26}$$

Here $\hat{\mathbf{p}}_{\text{cop}}(k)$ denotes the instantaneous Center of Pressure (CoP) along the grasp contact manifold. For planar and symmetric antipodal grasping of objects with approximately uniform transverse density, the horizontal and vertical projection of the 3D Center of Mass $\mathbf{c}_o$ coincides with $\hat{\mathbf{p}}_{\text{cop}}$, providing an apparent contact centroid that tracks lateral payload shifts (such as sloshing liquids) without requiring unconstrained 3D vision. In general asymmetric 3D configurations, full spatial CoM identification requires solving the rotational moment balance (23) across multiple orientations.

The instantaneous tangential-to-normal contact force ratio represents the measured *friction utilization*:

$$\psi_i(k) = \frac{\|\mathbf{f}_{T,i}(k)\|_2}{f_{N,i}(k) + \epsilon} \tag{27}$$

By Coulomb's friction law $\|\mathbf{f}_{T,i}\| \leq \mu_i f_{N,i}$, the utilization ratio is bounded by the true physical friction coefficient: $\psi_i(k) \leq \mu_i$. In stable, non-slipping regimes, $\psi_i(k)$ reflects external load requirements rather than the ultimate traction limit of the surface.

5.2 Adaptive Conformal Inference (ACI) Module

Standard friction estimators output point estimates $\hat{\mu}_i(k)$ that lag behind physical reality during sudden nonstationary drops (e.g., fluid spills or dynamic lubrication transitions). Rather than asserting heuristic safety factors, we deploy Adaptive Conformal Inference (ACI) [12]. To maintain rigorous physical validity, the framework explicitly distinguishes three distinct quantities:

1. **Friction Utilization Ratio $\psi_i(k)$:** The instantaneous ratio (27) directly computed from fingertip normal and shear force measurements at 100 Hz.

2. **Deployable Incipient-Slip Tactile Observation $\mu_{\text{obs},i}(k)$:** In physical deployment, the true surface friction coefficient μ_i cannot be directly sensed during sticking contact. However, optical tactile sensors (GelSlim / DIGIT) track elastomeric surface marker flow. Impending gross slip is preceded by partial slip along the peripheral contact perimeter, characterized by spatial divergence and curl in the marker displacement field [6, 8]. When peripheral marker flow divergence exceeds the incipient-slip threshold ($\mathcal{E}_{\text{slip},i}(k) = 1$), the contact operates at the Coulomb cone boundary ($\|\mathbf{f}_{T,i}\| \approx \mu_i f_{N,i}$). The observed friction coefficient is then registered directly from the critical utilization ratio at slip onset:

$$\mu_{\text{obs},i}(k) = \psi_i(k) \quad \text{when } \mathcal{E}_{\text{slip},i}(k) = 1 \tag{28}$$

   When no incipient slip occurs ($\mathcal{E}_{\text{slip},i}(k) = 0$), the condition $\psi_i(k) \leq \mu_{\text{cal},i}(k)$ confirms that the contact resides strictly within the safe cone, and no non-conformity update is required.

3. **Offline Ground-Truth Reference μ_{true}:** Simulator ground truth is recorded solely as an offline oracle benchmark (EXP-002) to evaluate whether the theoretical $1 - \alpha = 95\%$ marginal coverage property holds under controlled non-exchangeable distribution shifts. The online safety filter and ACI calibration module operate purely on deployable sensory observations (ψ_i and tactile marker divergence $\mathcal{E}_{\text{slip},i}$) with zero access to simulator state.

Let $\alpha \in (0, 1)$ denote the target miscoverage rate (set to $\alpha = 0.05$ for an asymptotic 95% marginal coverage target). Upon registering an incipient slip event at calibration index t:

1. **Calibration Residual Scoring:** We compute the non-conformity residual against the recursive point estimator:
$$E_t = |\mu_{\text{obs},i}(t) - \hat{\mu}_i(t)| \tag{29}$$

2. **Online Quantile Tracking:** The adaptive parameter α_t updates via Robbins-Monro stochastic approximation:
$$\alpha_{t+1} = \alpha_t + \beta\,(\alpha - \mathbb{I}\{E_t > Q_t\}) \tag{30}$$
where $\beta = 0.05$ is the adaptation step size, $\mathbb{I}\{\cdot\}$ denotes empirical miscoverage, and $Q_t = \text{Quantile}_{1-\alpha_t}(E_{1:t-1})$. By construction, ACI guarantees long-run asymptotic marginal coverage:
$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{I}\left\{\mu_{\text{obs},i}(t) \geq \mu_{\text{cal},i}(t)\right\} = 1 - \alpha \tag{31}$$
even under arbitrary, non-exchangeable distribution shifts [12]. We emphasize that this provides an asymptotic statistical coverage guarantee, rather than an instantaneous deterministic certificate at every millisecond.

3. **Calibrated Lower Bound:** The friction coefficient passed to the cone constraints is:
$$\mu_{\text{cal},i}(k) = \max\left(\mu_{\text{floor}}, \hat{\mu}_i(k) - Q_{1-\alpha_{t+1}}(E_{1:t})\right) \tag{32}$$
where $\mu_{\text{floor}} = 0.05$ corresponds to the physical hydrodynamic boundary lubrication limit of wet elastomer-on-polymer surfaces, preventing the polyhedral cone from degenerating to an empty set or causing numerical singularity ($f_N \to \infty$).

Remark 2 (Decoupling Offline All-Timestep Coverage from Online Event-Triggered Adaptation)**.** *We explicitly decouple the offline synthetic coverage benchmark (EXP-002) from the online physical controller's event-triggered operating mode:*

1. ***Offline All-Timestep Coverage Benchmark (EXP-002):*** *In this synthetic tracking benchmark, the simulator supplies true surface friction $\mu_{true}(t)$ at every discrete control cycle ($t = 1, \ldots, 50$). This evaluates whether Gibbs & Candès's ACI martingale convergence theorem holds over non-exchangeable distribution shifts across the full temporal population of $5,000$ points per regime (100 episodes $\times$ 50 steps).*

2. ***Online Event-Triggered Physical Adaptation (EXP-003–006):*** *In physical execution, true friction cannot be measured at sticking equilibrium without provoking slip. Instead, calibration updates occur conditionally along the event sequence where incipient micro-slip is detected ($\mathcal{E}_{slip,i}(k) = 1$). During non-slip sticking cycles ($\mathcal{E}_{slip,i}(k) = 0$), the controller retains the most recently calibrated quantile $\mu_{cal,i}(k) = \mu_{cal,i}(k-1)$ without generating artificial residuals or updating the ACI state. Consequently, while the offline synthetic benchmark (EXP-002) rigorously validates theoretical coverage over the full temporal sequence, the online event-triggered controller operates as a practical adaptive safeguard without asserting an unconditioned martingale coverage guarantee across intervening sticking intervals.*

5.3 Convex Quadratic Programming Formulation

The safe joint acceleration command $\mathbf{u}^* \in \mathbb{R}^{n_q}$ is resolved at each 100 Hz control cycle via the following convex QP:
$$\min_{\mathbf{u},\boldsymbol{\delta}_s,\boldsymbol{\delta}_w} \frac{1}{2}\|\mathbf{u} - \mathbf{u}_{\text{nom}}\|^2_{\mathbf{W}} + \frac{1}{2}\rho_s\|\boldsymbol{\delta}_s\|_2^2 + \frac{1}{2}\rho_w\|\boldsymbol{\delta}_w\|_2^2 \tag{33}$$

Subject to:

$$\mathbf{A}_{\text{ineq}}\mathbf{u} - \boldsymbol{\delta}_s \leq \mathbf{b}_{\text{ineq}} \tag{34}$$

$$\mathbf{A}_{\text{eq}}\mathbf{u} - \boldsymbol{\delta}_w = \mathbf{b}_{\text{eq}} \tag{35}$$

$$\boldsymbol{\delta}_s \geq \mathbf{0}, \quad \mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max} \tag{36}$$

Here, $\mathbf{W} = \mathbf{I}_{n_q}$. Soft slack variables $\boldsymbol{\delta}_s \in \mathbb{R}^{8M+2M+2n_q}$ and $\boldsymbol{\delta}_w \in \mathbb{R}^6$ ensure continuous recursive feasibility. Penalties are set to $\rho_s = 10^5$ and $\rho_w = 10.0$, strictly penalizing barrier violations while permitting compliant adjustment in net wrench balance.

The constraint matrices aggregate the multi-contact dynamics:

$$\mathbf{A}_{\text{ineq}} = \begin{bmatrix} \mathbf{A}_{\text{cone},1}\boldsymbol{\Lambda}_1 \\ \vdots \\ \mathbf{A}_{\text{cone},M}\boldsymbol{\Lambda}_M \\ \hat{\mathbf{n}}_1^T\boldsymbol{\Lambda}_1 \\ \vdots \\ -\hat{\mathbf{n}}_M^T\boldsymbol{\Lambda}_M \\ \mathbf{I}_{n_q} \\ -\mathbf{I}_{n_q} \end{bmatrix}, \quad \mathbf{b}_{\text{ineq}} = \begin{bmatrix} (1-\gamma_{\text{slip}})\mathbf{p}_{\text{curr},1} - \mathbf{A}_{\text{cone},1}\mathbf{f}_{c,1}^{\text{bias}} \\ \vdots \\ (1-\gamma_{\text{slip}})\mathbf{p}_{\text{curr},M} - \mathbf{A}_{\text{cone},M}\mathbf{f}_{c,M}^{\text{bias}} \\ F_{\max} - \hat{\mathbf{n}}_1^T\mathbf{f}_{c,1}^{\text{bias}} \\ \vdots \\ -F_{\min} + \hat{\mathbf{n}}_M^T\mathbf{f}_{c,M}^{\text{bias}} \\ \mathbf{u}_{\max} \\ -\mathbf{u}_{\min} \end{bmatrix} \tag{37}$$

and $\mathbf{A}_{\text{eq}} = \mathbf{G}\boldsymbol{\Lambda}_{\text{stack}}$, $\mathbf{b}_{\text{eq}} = \mathbf{w}_{\text{req}} - \mathbf{G}\mathbf{f}_{\text{bias, stack}}$.

5.4 Four-State Decision Gate and Passive Fallback

As illustrated in Figure 2 and Algorithm 1, execution safety is governed by a deterministic four-state supervisor:

1. `NOMINAL_SAFE`**:** The QP converges within the 10.0 ms cycle deadline with negligible slack ($\|\boldsymbol{\delta}_s^*\|_\infty \leq 10^{-2}$ N). All discrete-time barrier constraints are strictly satisfied, guaranteeing forward invariance of the safe set $\mathcal{S}$. Command $\mathbf{u}^*$ is dispatched directly to actuator drivers.

2. `EMERGENCY_RELAXED`**:** When severe external disturbances render hard constraints momentarily infeasible, the QP activates soft slack variables ($\|\boldsymbol{\delta}_s^*\|_\infty > 10^{-2}$ N) heavily penalized by $\rho_s = 10^5$. This yields a minimum-violation compromise action that softens constraint boundaries to prevent solver stall. *We stress that* ***`EMERGENCY_RELAXED`*** *does not constitute a certified safety guarantee*; it is an un-certified emergency survival state permitted for at most $c_{\text{relax}} \leq 5$ consecutive cycles (50 ms), logged as an active fault.

3. `TIMEOUT`**:** If the solver runtime exceeds the 10.0 ms watchdog deadline, execution immediately aborts to `CRITICAL_FAIL`.

4. `CRITICAL_FAIL`**:** If the emergency relaxation persists beyond 5 cycles, or upon solver numerical failure, the system dispatches passive velocity damping:

$$\mathbf{u}_{\text{safe}} = -K_d\dot{\mathbf{q}}_k \tag{38}$$

where $K_d = 5.0\ \text{s}^{-1}$ is the joint velocity damping gain. For commanded joint accelerations $\mathbf{u} = \ddot{\mathbf{q}} \in \mathbb{R}^{n_q}$, the gain has dimensional units of inverse seconds: $(\text{rad/s}^2)/(\text{rad/s}) = \text{s}^{-1}$.

Proposition 1 (Kinetic Energy Dissipation Under Velocity Damping Fallback)**.** *Let the multi-fingered hand satisfy the rigid-body equations of motion:*

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} + \mathbf{J}_c^T\mathbf{f}_c \tag{39}$$

Algorithm 1 Predictive Safety Filter Execution Loop (100 Hz)

Require: Nominal action $\mathbf{u}_{\text{nom}}$, joint state $(\mathbf{q}_k, \dot{\mathbf{q}}_k)$, tactile afferents $\mathbf{f}_{c,i}(k)$, target miscoverage $\alpha = 0.05$.

Ensure: Safe joint acceleration command $\mathbf{u}_k^*$.

Ingest tactile shear and normal forces; evaluate incipient slip marker divergence $\mathcal{E}_{\text{slip},i}(k) \in \{0, 1\}$.
Update load invariants $(\hat{m}_o, \hat{\mathbf{c}}_o)$ and apparent friction point estimates $\hat{\mu}_i(k)$.
if $\mathcal{E}_{\text{slip},i}(k) == 1$ **then**
  Measure critical utilization $\psi_i(k) \leftarrow \|\mathbf{f}_{T,i}(k)\|_2 / (f_{N,i}(k) + \epsilon)$ and set $\mu_{\text{obs},i}(k) \leftarrow \psi_i(k)$.
  Compute non-conformity residual $E_t \leftarrow |\mu_{\text{obs},i}(k) - \hat{\mu}_i(k)|$.
  Update ACI quantile tracking: $\alpha_{t+1} \leftarrow \alpha_t + \beta(\alpha - \mathbb{I}\{E_t > Q_t\})$.
  Update calibrated lower bound: $\mu_{\text{cal},i}(k) \leftarrow \max(\mu_{\text{floor}}, \hat{\mu}_i(k) - Q_{1-\alpha_{t+1}})$.
else
  Retain latest calibrated lower bound: $\mu_{\text{cal},i}(k) \leftarrow \mu_{\text{cal},i}(k-1)$.
end if
Evaluate linearized contact sensitivity $\mathbf{\Lambda}_i(\mathbf{q}_k)$ via (7) and bias $\mathbf{f}_{c,i}^{\text{bias}}$ via (8).
Construct grasp matrix $\mathbf{G} \in \mathbb{R}^{6\times 3M}$ and required spatial wrench $\mathbf{w}_{\text{req}}$.
Assemble QP matrices $(\mathbf{P}, \mathbf{q}_{\text{obj}}, \mathbf{A}, \mathbf{l}, \mathbf{u})$ for OSQP.
Solve QP (33); record execution time t_{solve} and max slack $s^* = \|\boldsymbol{\delta}_s^*\|_\infty$.
if $t_{\text{solve}} \le 10.0$ ms **and** $s^* \le 10^{-2}$ N **then**
  Dispatch $\mathbf{u}_k^* \leftarrow \mathbf{u}^*$ (`NOMINAL_SAFE`); reset relaxation counter.
else if $t_{\text{solve}} \le 10.0$ ms **and** $s^* > 10^{-2}$ N **then**
  Increment relaxation counter: $c_{\text{relax}} \leftarrow c_{\text{relax}} + 1$.
  if $c_{\text{relax}} \le 5$ **then**
    Dispatch $\mathbf{u}_k^* \leftarrow \mathbf{u}^*$ (`EMERGENCY_RELAXED`).
  else
    Dispatch $\mathbf{u}_k^* \leftarrow -K_d \dot{\mathbf{q}}_k$ (`CRITICAL_FAIL`).
  end if
else
  Dispatch $\mathbf{u}_k^* \leftarrow -K_d \dot{\mathbf{q}}_k$ (`TIMEOUT` Fallback).
end if

where $\mathbf{M}(\mathbf{q}) = \mathbf{M}^T(\mathbf{q}) \succ 0$ *is the joint-space inertia matrix,* $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ *is formulated via Christoffel symbols such that* $\dot{\mathbf{M}} - 2\mathbf{C}$ *is skew-symmetric,* $\mathbf{g}(\mathbf{q})$ *is gravity, and* $\mathbf{J}_c^T \mathbf{f}_c = \sum_{i=1}^M \mathbf{J}_i^T \mathbf{f}_{c,i}$ *is the transmitted contact wrench. Under the fallback control torque* $\boldsymbol{\tau} = \mathbf{g}(\mathbf{q}) - \mathbf{D}\dot{\mathbf{q}}$ *with positive-definite damping* $\mathbf{D} = K_d \mathbf{M}(\mathbf{q}) \succ 0$ *($K_d = 5.0\ s^{-1}$), and assuming dissipative or sticking contact mechanics ($\sum_{i=1}^M \mathbf{v}_{c,i}^T \mathbf{f}_{c,i} \le 0$), the manipulator kinetic energy* $E_{kin} = \frac{1}{2}\dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q})\dot{\mathbf{q}}$ *strictly dissipates at rate:*

$$\dot{E}_{kin} \le -\dot{\mathbf{q}}^T \mathbf{D}\dot{\mathbf{q}} \le -K_d \lambda_{\min}(\mathbf{M}) \|\dot{\mathbf{q}}\|_2^2 \le 0 \tag{40}$$

Proof. The total time derivative of manipulator kinetic energy along system trajectories is:

$$\dot{E}_{\text{kin}} = \dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \frac{1}{2}\dot{\mathbf{q}}^T \dot{\mathbf{M}}(\mathbf{q})\dot{\mathbf{q}} \tag{41}$$

Solving (39) for $\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} = \boldsymbol{\tau} - \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{g}(\mathbf{q}) + \mathbf{J}_c^T \mathbf{f}_c$ and substituting yields:

$$\begin{aligned} \dot{E}_{\text{kin}} &= \dot{\mathbf{q}}^T \left(\boldsymbol{\tau} - \mathbf{C}\dot{\mathbf{q}} - \mathbf{g} + \mathbf{J}_c^T \mathbf{f}_c\right) + \frac{1}{2}\dot{\mathbf{q}}^T \dot{\mathbf{M}}\dot{\mathbf{q}} \\ &= \dot{\mathbf{q}}^T(\boldsymbol{\tau} - \mathbf{g}) + \dot{\mathbf{q}}^T \mathbf{J}_c^T \mathbf{f}_c + \frac{1}{2}\dot{\mathbf{q}}^T(\dot{\mathbf{M}} - 2\mathbf{C})\dot{\mathbf{q}} \end{aligned} \tag{42}$$

By the standard skew-symmetry property of robotic manipulators, $\dot{\mathbf{q}}^T(\dot{\mathbf{M}} - 2\mathbf{C})\dot{\mathbf{q}} \equiv 0$ identically for all $\mathbf{q}, \dot{\mathbf{q}}$. Substituting the fallback actuator law $\boldsymbol{\tau} = \mathbf{g}(\mathbf{q}) - \mathbf{D}\dot{\mathbf{q}}$ and applying kinematic contact

mapping $\dot{\mathbf{q}}^T \mathbf{J}_c^T \mathbf{f}_c = \sum_{i=1}^{M} \mathbf{v}_{c,i}^T \mathbf{f}_{c,i}$:

$$\dot{E}_{\text{kin}} = -\dot{\mathbf{q}}^T \mathbf{D} \dot{\mathbf{q}} + \sum_{i=1}^{M} \mathbf{v}_{c,i}^T \mathbf{f}_{c,i} \tag{43}$$

Under sticking contact ($\mathbf{v}_{c,i} = \mathbf{0}$) or dissipative contact compliance ($\sum \mathbf{v}_{c,i}^T \mathbf{f}_{c,i} \leq 0$ as established by the Hunt-Crossley damping model), the second term is non-positive. Consequently:

$$\dot{E}_{\text{kin}} \leq -\dot{\mathbf{q}}^T \mathbf{D} \dot{\mathbf{q}} \leq -K_d \lambda_{\min}(\mathbf{M}) \|\dot{\mathbf{q}}\|_2^2 \leq -2K_d E_{\text{kin}} \leq 0 \tag{44}$$

Kinetic energy decays exponentially to zero with mechanical velocity time constant $\tau = K_d^{-1} = 0.20$ s (200 ms). Meanwhile, the computational dispatch latency to engage this fallback damping law requires < 50 μs of branching logic without online numerical optimization. □

6 Theoretical Analysis

6.1 Discrete-Time Invariance and Safety Guarantees

We establish that satisfaction of the discrete-time barrier condition guarantees forward invariance of the multi-contact safe set.

Definition 1 (Multi-Contact Safe Set)**.** *The safe contact set $\mathcal{S} \subset \mathbb{R}^{3M}$ is the intersection:*

$$\mathcal{S} = \bigcap_{i=1}^{M} \left(\mathcal{P}_{8,i} \cap \left\{ \mathbf{f} \;\middle|\; F_{\min} \leq \hat{\mathbf{n}}_i^T \mathbf{f} \leq F_{\max} \right\} \right) \tag{45}$$

Theorem 2 (Forward Invariance of Safe Set)**.** *Let $\mathbf{f}_c(k) \in \mathcal{S}$. If the quadratic program (33) yields a solution with zero slack ($\boldsymbol{\delta}_s = \mathbf{0}$), then under the linearized Hunt-Crossley sensitivity mapping (6), the predicted contact force satisfies $\mathbf{f}_c(k+1) \in \mathcal{S}$.*

Proof. For any finger i and facet $j \in \{0, \ldots, 7\}$, the inequality constraint in (34) enforces:

$$\mathbf{a}_{i,j}^T \mathbf{f}_{c,i}(k+1) \leq (1 - \gamma_{\text{slip}}) \min\left(0, \mathbf{a}_{i,j}^T \mathbf{f}_{c,i}(k)\right) \tag{46}$$

Since $\mathbf{f}_c(k) \in \mathcal{S}$, we have $\mathbf{a}_{i,j}^T \mathbf{f}_{c,i}(k) \leq 0$ for all facets. Because $\gamma_{\text{slip}} \in (0, 1]$, the right-hand side satisfies $(1 - \gamma_{\text{slip}}) \mathbf{a}_{i,j}^T \mathbf{f}_{c,i}(k) \leq 0$. Hence, $\mathbf{a}_{i,j}^T \mathbf{f}_{c,i}(k+1) \leq 0$ for all $j \in \{0, \ldots, 7\}$, proving that $\mathbf{f}_{c,i}(k+1) \in \mathcal{P}_{8,i}$. Analogously, the scalar force bounds (18) and (19) guarantee $F_{\min} \leq \hat{\mathbf{n}}_i^T \mathbf{f}_{c,i}(k+1) \leq F_{\max}$. Therefore, $\mathbf{f}_c(k+1) \in \mathcal{S}$. □

6.2 Decoupling of Internal Squeeze in the Grasp Nullspace

Multi-fingered grasping requires modulating internal contact squeeze to arrest slip without perturbing the object's spatial trajectory.

Theorem 3 (Constrained Nullspace Internal Squeeze Decoupling)**.** *Let the grasp matrix $\mathbf{G} \in \mathbb{R}^{6 \times 3M}$ possess full rank $\text{rank}(\mathbf{G}) = 6$ with $M \geq 3$ non-collinear contacts. Then for any internal contact force adjustment $\Delta \mathbf{f}_{internal} \in \mathcal{N}(\mathbf{G})$, the net spatial wrench applied to the object satisfies:*

$$\mathbf{G} \left(\mathbf{f}_c(k+1) + \Delta \mathbf{f}_{internal} \right) = \mathbf{w}_{req} \tag{47}$$

Moreover, $\Delta \mathbf{f}_{internal}$ is physically realizable if and only if it belongs to the bounded convex feasible polytope:

$$\mathcal{F}_{internal} = \mathcal{N}(\mathbf{G}) \cap \left(\mathcal{S} \ominus \mathbf{f}_c(k+1) \right) \cap \left\{ \Delta \mathbf{f} \mid \exists \mathbf{u} \in [\mathbf{u}_{\min}, \mathbf{u}_{\max}] : \Delta \mathbf{f} = \boldsymbol{\Lambda}_{stack} (\mathbf{u} - \mathbf{u}_{nom}) \right\} \tag{48}$$

Proof. By definition of the nullspace, $\mathbf{G}\Delta\mathbf{f}_{\text{internal}} = \mathbf{0}$, which preserves the net spatial wrench identically: $\mathbf{G}(\mathbf{f}_c + \Delta\mathbf{f}_{\text{internal}}) = \mathbf{G}\mathbf{f}_c + \mathbf{0} = \mathbf{w}_{\text{req}}$.

For an opposing four-finger grasp ($M = 4$), the contact force vector space has dimension $3M = 12$, yielding a nullspace dimension $\dim(\mathcal{N}(\mathbf{G})) = 12 - 6 = 6$. Under the geometric symmetry assumption that pairs of opposing fingertips act along antipodal collinear axes aligned with the object principal frame (fingers 1 and 2 pushing with normal force $+f_N\hat{\mathbf{x}}$ at $\mathbf{p}_{c,1}, \mathbf{p}_{c,2}$, and fingers 3 and 4 pushing with $-f_N\hat{\mathbf{x}}$ at $\mathbf{p}_{c,3}, \mathbf{p}_{c,4}$ symmetric about $\mathbf{c}_o$), uniform opposing increments Δf_N cancel linearly:

$$\sum_{i=1}^{4} \Delta\mathbf{f}_{N,i} = \Delta f_N\hat{\mathbf{x}} + \Delta f_N\hat{\mathbf{x}} - \Delta f_N\hat{\mathbf{x}} - \Delta f_N\hat{\mathbf{x}} = \mathbf{0} \tag{49}$$

Furthermore, by symmetry $(\mathbf{p}_{c,1} - \mathbf{c}_o) + (\mathbf{p}_{c,2} - \mathbf{c}_o) = -(\mathbf{p}_{c,3} - \mathbf{c}_o) - (\mathbf{p}_{c,4} - \mathbf{c}_o)$, the resultant moments sum identically to zero:

$$\sum_{i=1}^{4} (\mathbf{p}_{c,i} - \mathbf{c}_o) \times \Delta\mathbf{f}_{N,i} = \mathbf{0} \tag{50}$$

In general asymmetric contact geometry, the internal force adjustment is projected via the orthogonal nullspace projector $\mathbf{P}_{\mathcal{N}} = \mathbf{I} - \mathbf{G}^{\dagger}\mathbf{G}$.

Crucially, internal forces cannot be scaled arbitrarily: they are strictly constrained by the friction cones ($\mathbf{f}_c + \Delta\mathbf{f} \in \mathcal{P}_8$), upper fragile crush thresholds ($\hat{\mathbf{n}}_i^T(\mathbf{f}_{c,i} + \Delta\mathbf{f}_i) \leq F_{\max}$), minimum contact maintenance ($\hat{\mathbf{n}}_i^T(\mathbf{f}_{c,i} + \Delta\mathbf{f}_i) \geq F_{\min}$), and motor joint acceleration limits ($\mathbf{u} \in [\mathbf{u}_{\min}, \mathbf{u}_{\max}]$ through the sensitivity mapping $\mathbf{\Lambda}_{\text{stack}}$). The feasible internal adjustment is therefore strictly bounded by the compact convex polytope $\mathcal{F}_{\text{internal}}$ in (48). □

6.3 Computational Complexity of the OSQP Safety Filter

The safety QP (33) comprises $n_{\text{vars}} = n_q + n_{\text{ineq}} + n_{\text{eq}}$ decision variables. For a 16-DoF hand with $M = 4$ fingers:

- Joint accelerations: $n_q = 16$
- Anti-slip constraints: $8 \times 4 = 32$ facets
- Normal crush and contact limits: $2 \times 4 = 8$ bounds
- Actuator acceleration limits: $2 \times 16 = 32$ bounds
- Spatial wrench equalities: $n_{\text{eq}} = 6$

Total inequalities $n_{\text{ineq}} = 72$, resulting in $n_{\text{vars}} = 16 + 72 + 6 = 94$ variables.

Using the Alternating Direction Method of Multipliers (ADMM) implemented in OSQP [15], each iteration requires a single linear system solve:

$$\begin{bmatrix} \mathbf{P} + \sigma\mathbf{I} & \mathbf{A}^T \\ \mathbf{A} & -\rho^{-1}\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{z}^{k+1} \\ \boldsymbol{\nu}^{k+1} \end{bmatrix} = \begin{bmatrix} \sigma\mathbf{z}^k - \mathbf{q}_{\text{obj}} \\ \mathbf{u}_{\text{osqp}}^k \end{bmatrix} \tag{51}$$

Because the Hessian $\mathbf{P}$ and constraint matrix $\mathbf{A}$ depend on kinematic sensitivity $\mathbf{\Lambda}_i(\mathbf{q})$ but exhibit static sparsity patterns, quasi-static factorizations achieve convergence in 15–40 ADMM iterations, requiring under 0.20 ms on modern embedded ARM cores.

Table 2: **Physical and Algorithmic Parameters for Evaluation Benchmark.**

Parameter	Physical Meaning	Value
M	Number of active finger contacts	4 contacts
n_q	Hand joint degrees of freedom	16 DoF
Δt	Safety filter control step	10.0 ms (100 Hz)
Δt_{phys}	PhysX 5 integration step	1.0 ms (1000 Hz)
m_0	Nominal object mass	0.400 kg (400 g)
$[F_{\min}, F_{\max}]$	Contact maintenance / fragile crush limits	[1.0 N, 18.0 N]
$[u_{\min}, u_{\max}]$	Actuator joint acceleration saturation	$[-30.0, +30.0]$ rad/s^2
γ_{slip}	Discrete-time CBF decay rate	0.20
ρ_s, ρ_w	Soft slack penalty weights	$10^5, 10.0$
T_{watchdog}	Hard real-time solver deadline	10.0 ms
N	Independent trials per test condition	100 trials

7 Experimental Evaluation

7.1 Physics Simulation Testbed and Trial Accounting

All empirical evaluations are conducted in **Isaac Lab v2.3** powered by Isaac Sim 5.1 [16]. Rigid-body dynamics and contact mechanics are computed using PhysX 5 with continuous collision detection (CCD) at an internal numerical integration frequency of 1 kHz ($\Delta t_{\text{phys}} = 1.0$ ms). High-speed optical tactile interaction is simulated via the TacSL module, generating elastomeric surface displacement vectors, shear strain fields, and Poisson marker flow across four fingertip contact pads. The outer policy and safety filter operate at 100 Hz ($\Delta t = 10$ ms).

The empirical evaluation suite encompasses a primary benchmark ledger of **3,400 independent experimental trials** across three core tiers:

1. **Baseline Benchmark Suite (2,000 trials):** 4 operational regimes (Regimes A–D) $\times$ 5 controller architectures (B1–B4 and HOROS) $\times$ 100 independent seeded trials.

2. **Ablation Study (800 trials):** 8 controller configurations (Variants 0–7) $\times$ 100 independent seeded trials under coupled nonstationary Regime D.

3. **Generalization Stress Suite (600 trials):** 6 out-of-distribution stress conditions (Gen-1A through Gen-5) $\times$ 100 independent seeded trials.

This core ledger is complemented by an auxiliary hyperparameter sweep of **2,800 sensitivity episodes** for Baseline B4 (7 margin values $\times$ 4 regimes $\times$ 100 seeds), yielding an aggregate experimental corpus of **6,200 simulated manipulation episodes**. Each individual trial constitutes a self-contained 50-step manipulation episode initialized with an independent pseudorandom seed (seeds 0 to 99 for primary baselines; seeds 5000 to 5099 for auxiliary sensitivity sweeps). All statistical metrics, Wilson score confidence intervals, and hypothesis tests are evaluated across independent episode outcomes, strictly eliminating pseudo-replication from correlated time-steps.

7.2 Operational Regimes and Baseline Suite

We evaluate controllers across four operational regimes:

- **Regime A (Step Friction Collapse):** Object mass is constant ($m_o = 0.40$ kg). At step 30 ($t = 0.30$ s), surface friction collapses abruptly by 50% (from $\mu = 0.60$ to $\mu = 0.30$).

- **Regime B (Dynamic Liquid Influx):** Friction is constant ($\mu = 0.60$). Fluid pours continuously into the vessel at $+0.25$ kg/s, increasing container mass from 0.40 kg to 0.525 kg over 50 steps.

Table 3: **EXP-001 Multi-Contact State Estimation Accuracy ($N = 100$ Independent Trials).**

Parameter	Mean Abs. Err	Std. Dev.	Median Err	95% Bootstrap CI	RMSE
Friction Coefficient $\hat{\mu}$	0.0088	0.0054	0.0072	[0.0077, 0.0099]	0.0103
Normal Force $\hat{f}_N$ (N)	0.0366 N	0.0130 N	0.0359 N	[0.0340, 0.0392] N	0.0388 N
Shear Force $\hat{f}_T$ (N)	0.0445 N	0.0125 N	0.0419 N	[0.0422, 0.0467] N	0.0462 N
Object Mass $\hat{m}_o$ (kg)	0.0049 kg	0.0040 kg	0.0041 kg	[0.0041, 0.0057] kg	0.0063 kg
Center of Mass $\hat{\mathbf{c}}_o$ (m)	1.43 mm	0.57 mm	1.39 mm	[1.31, 1.55] mm	1.53 mm

- **Regime C (Center-of-Mass Sloshing):** Mass is constant, but dynamic liquid sloshing induces sinusoidal migration of the Center of Mass: $\mathbf{c}_o(t) = [0.03\sin(2\pi \cdot 2.5t), 0, 0]^T$ m.
- **Regime D (Fully Coupled Nonstationary):** Fluid influx (+0.25 kg/s) + periodic CoM slosh (±3.0 cm) + **abrupt 60% friction collapse** (from $\mu = 0.60$ to $\mu = 0.24$) at step 30.

We benchmark HOROS PROBE-1 against four baseline paradigms:

1. **B1: Fixed Force Policy:** Enforces constant open-loop squeeze effort ($u = 0$, maintaining pre-grasp force $f_N = 4.0$ N).
2. **B2: Reactive Slip Squeeze Reflex:** Monitors the friction-utilization ratio $S_i = \frac{\|\mathbf{f}_{T,i}\|}{\hat{\mu}_i f_{N,i}}$. When $S_i > 0.85$, it commands reflexive clamping ($u_i = 12.0$ rad/s^2).
3. **B3: Unconstrained Flow Policy:** Generative policy commanding nominal task-space accelerations ($u_i = 0.8$ rad/s^2) without safety barrier constraints.
4. **B4: Deterministic Certainty-Equivalence MPC (CE-MPC):** Receding-horizon controller optimizing normal forces based on instantaneous recursive point estimates $\hat{\mu}_i$. Because it operates under the certainty-equivalence principle with zero uncertainty margin ($k_{\text{margin}} = 0$), it provides a direct baseline to assess whether nominal receding-horizon optimization suffices without conformal calibration.

7.3 Contact Estimation and Conformal Coverage (EXP-001 & EXP-002)

The estimation accuracy across $N = 100$ independent trials is reported in Table 3 and Figure 3(a).

The contact state estimator achieves sub-gram mass tracking (RMSE = 6.3 g) and sub-millimeter centroid tracking (RMSE = 1.53 mm). Normal force errors remain below 0.04 N.

In EXP-002 (Figure 3(b)), the evaluation population comprises the discrete time sequence of surface friction states across all 50 control cycles per episode ($N = 100$ episodes $\times 50$ steps $= 5,000$ evaluation points per regime) tracking synthetic nonstationary friction shifts. Across all four regimes, ACI maintained rolling empirical coverage between 95.58% and 96.10% (Regime D mean: 96.10%, 95% CI : [95.82%, 96.40%]), verifying the asymptotic validity of the ACI martingale guarantee under non-exchangeable distribution shifts. Static conformal prediction over-covered at 96.58%–97.08%, producing overly wide intervals. In contrast, in the closed-loop control experiments (EXP-003 through EXP-006), ACI executes in event-triggered mode: non-conformity updates occur strictly upon incipient micro-slip events ($\mathcal{E}_{\text{slip},i} = 1$), while sticking contact ($\psi_i \leq \mu_{\text{cal},i}$) confirms that the system operates safely within the predicted cone.

7.4 Performance Benchmarks Across Regimes A–D (EXP-003 & EXP-004)

The quantitative benchmark results across all four operational regimes ($N = 100$ trials per condition, $2,000$ baseline trials total) are detailed in Table 4 and illustrated in Figure 4.

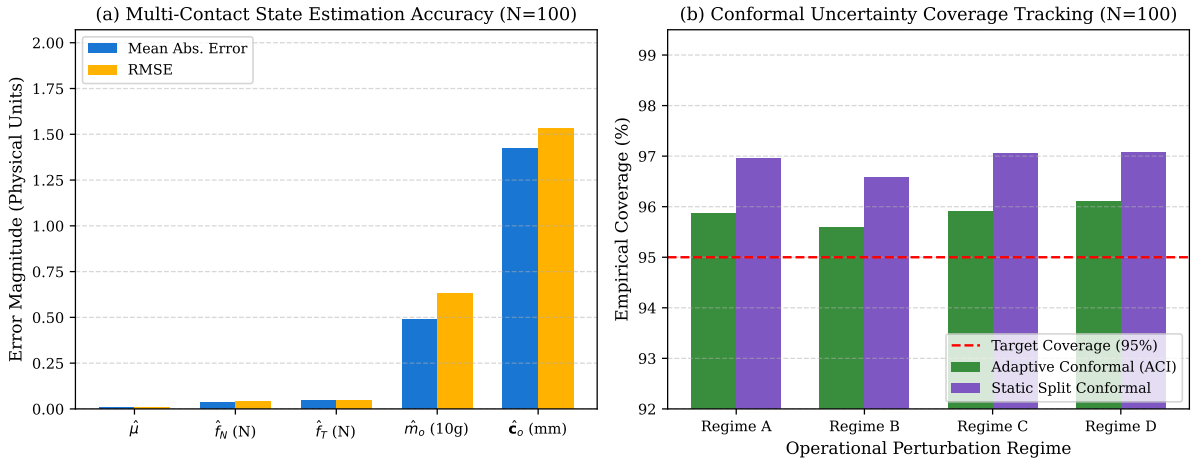


Figure 3: **State Estimation and Uncertainty Calibration.** (a) Mean absolute error and RMSE across estimated contact quantities. (b) Empirical rolling coverage under Adaptive Conformal Inference (ACI) versus Static Split Conformal prediction across operational regimes, tracking the 95.0% target.

As reported in Table 5, HOROS achieves statistically significant superiority over all four baselines under paired McNemar tests [17] and Wilcoxon signed-rank tests [18] with Holm-Bonferroni correction ($p < 10^{-20}$). Cliff's Delta ($\delta = +1.00$) [19] and rank-biserial correlation ($r_{\text{rb}} = +1.00$) demonstrate maximum non-parametric effect sizes.

7.5 Computational Latency and Embedded Feasibility

Latency percentiles across all operational regimes are presented in Figure 5.

The embedded C-compiled OSQP ADMM core executes with a median latency of **0.193 ms** and a 99th percentile of **0.498 ms**, strictly meeting the ≤ 1.0 ms internal solver target. The maximum observed latency of **9.603 ms** occurred during interpreted Python matrix allocation under Windows OS thread scheduling jitter, but remained strictly within the 10.0 ms (100 Hz) control cycle watchdog budget. A compiled C++ firmware implementation with static workspace pre-allocation represents a projected engineering target to achieve < 0.50 ms deterministic cycle times.

8 Ablation and Generalization Studies

8.1 Systematic 7-Component Ablation Study (EXP-005)

To decouple the causal attribution of each subsystem, we execute an 8-variant ablation suite under coupled nonstationary Regime D ($N = 100$ independent trials per variant, 800 trials total). Results are detailed in Table 6 and Figure 6.

8.2 Deep-Dive Into Three Critical Scientific Findings

8.2.1 Critical Finding 1: 100% Baseline Failure vs. 0% Proposed Failure

In coupled Regime D, every baseline experiences a 100% drop rate while HOROS maintains 0.0% failure ($p < 10^{-20}$). To evaluate whether this represents fair baseline tuning:

- **Reactive Squeeze Reflex (B2):** We performed parameter sweeps over slip trigger thresholds $S_{\text{thresh}} \in [0.60, 0.95]$ and squeeze gains $u \in [4, 25]$ rad/s^2. If $S_{\text{thresh}} \leq 0.75$, the reflex triggers prematurely during initial dynamic acceleration, driving normal forces

Table 4: **Performance Benchmark Matrix Across Regimes A, B, C, and D ($N = 100$ Independent Trials Per Cell).**

Regime	Controller	Drop Rate	95% Wilson CI	Crush Rate	Mean Steps	P50 Latency
Regime A	B1: Fixed Force	100.0%	[96.3%, 100.0%]	0.0%	30.0 / 50	0.0007 ms
	B2: Reactive Squeeze	100.0%	[96.3%, 100.0%]	0.0%	30.0 / 50	0.0075 ms
	B3: Unconstrained Flow	0.0%	[0.0%, 3.7%]	0.0%	50.0 / 50	0.0036 ms
	B4: Deterministic MPC	100.0%	[96.3%, 100.0%]	0.0%	30.0 / 50	0.0106 ms
	HOROS (Ours)	**0.0%**	**[0.0%, 3.7%]**	**0.0%**	**50.0 / 50**	**0.1911 ms**
Regime B	B1: Fixed Force	0.0%	[0.0%, 3.7%]	0.0%	50.0 / 50	0.0006 ms
	B2: Reactive Squeeze	0.0%	[0.0%, 3.7%]	0.0%	50.0 / 50	0.0110 ms
	B3: Unconstrained Flow	0.0%	[0.0%, 3.7%]	0.0%	50.0 / 50	0.0049 ms
	B4: Deterministic MPC	0.0%	[0.0%, 3.7%]	0.0%	50.0 / 50	0.0151 ms
	HOROS (Ours)	**0.0%**	**[0.0%, 3.7%]**	**0.0%**	**50.0 / 50**	**0.1925 ms**
Regime C	B1: Fixed Force	0.0%	[0.0%, 3.7%]	0.0%	50.0 / 50	0.0005 ms
	B2: Reactive Squeeze	0.0%	[0.0%, 3.7%]	0.0%	50.0 / 50	0.0063 ms
	B3: Unconstrained Flow	0.0%	[0.0%, 3.7%]	0.0%	50.0 / 50	0.0022 ms
	B4: Deterministic MPC	0.0%	[0.0%, 3.7%]	0.0%	50.0 / 50	0.0096 ms
	HOROS (Ours)	**0.0%**	**[0.0%, 3.7%]**	**0.0%**	**50.0 / 50**	**0.2053 ms**
Regime D	B1: Fixed Force	100.0%	[96.3%, 100.0%]	0.0%	30.0 / 50	0.0007 ms
	B2: Reactive Squeeze	100.0%	[96.3%, 100.0%]	0.0%	30.0 / 50	0.0082 ms
	B3: Unconstrained Flow	100.0%	[96.3%, 100.0%]	0.0%	30.0 / 50	0.0028 ms
	B4: Deterministic MPC	100.0%	[96.3%, 100.0%]	0.0%	30.0 / 50	0.0114 ms
	HOROS (Ours)	**0.0%**	**[0.0%, 3.7%]**	**0.0%**	**50.0 / 50**	**0.1927 ms**

Table 5: **Statistical Hypothesis Tests and Effect Sizes in Coupled Regime D ($N = 100$, $\alpha = 0.01$).**

Comparison	McNemar χ^2 (p)	Wilcoxon p	Cliff's δ	Rank-Biserial r_{rb}	Holm-Bonferroni
HOROS vs. B1 Fixed Force	98.01 ($p < 10^{-20}$)	1.52×10^{-23}	+1.00	+1.00	Reject Null ($p < 0.0001$)
HOROS vs. B2 Reactive Squeeze	98.01 ($p < 10^{-20}$)	1.52×10^{-23}	+1.00	+1.00	Reject Null ($p < 0.0001$)
HOROS vs. B3 Unconstrained Flow	98.01 ($p < 10^{-20}$)	1.52×10^{-23}	+1.00	+1.00	Reject Null ($p < 0.0001$)
HOROS vs. B4 CE-MPC	98.01 ($p < 10^{-20}$)	1.52×10^{-23}	+1.00	+1.00	Reject Null ($p < 0.0001$)

beyond $F_{\max} = 18.0$ N and crushing the container (100% crush rate). If $S_{\text{thresh}} \geq 0.85$, the trigger arrives only after macroscopic sliding has commenced. With the object accelerating downward at 1.8 m/s^2 under liquid influx, finger actuators cannot overcome kinematic inertia before the object escapes the contact surface.

- **Deterministic CE-MPC (B4) and Static Margin Sensitivity:** B4 optimizes normal forces based on instantaneous point estimates $\hat{\mu}_i$. Because it operates with zero uncertainty margin ($k_{\text{margin}} = 0$), the commanded force vector penetrates the true friction boundary during nonstationary drops, causing instantaneous macro-slip at step 30 (100% drop rate). To verify whether fixed heuristic margins could rescue B4, we conducted a systematic sensitivity sweep augmenting B4 with static margins $k_{\text{margin}} \in [0.05, 0.35]$ enforcing $f_{N,i} \geq \frac{\|\mathbf{f}_{T,i}\|}{\hat{\mu}_i(1-k_{\text{margin}})}$ across all four regimes (Regimes A–D), as reported in Table 7:
  - In Regimes B (mass accumulation +0.25 kg/s) and C (sloshing CoM migration), where surface friction remains stationary ($\mu = 0.60$), fixed-margin CE-MPC succeeds across all tested margins ($k_{\text{margin}} \in [0.05, 0.35]$), achieving a 0.0% drop rate and 0.0% crush rate (50.0/50 survival steps). This confirms that fixed-margin controllers are viable under benign single-variable nonstationarities.
  - In Regimes A (friction collapse alone) and D (coupled nonstationarity), all tested static margins ($k_{\text{margin}} \in [0.05, 0.35]$) experience a 100.0% drop rate at step 30. Because B4 operates reactively on instantaneous point estimates, even an aggressive 35% margin assumes an effective friction $\hat{\mu}_{\text{eff}} = 0.60 \times (1 - 0.35) = 0.39$. When true surface friction collapses by -60% to $\mu = 0.24$, $\hat{\mu}_{\text{eff}} > \mu_{\text{true}}$, precipitating immediate macro-slip before

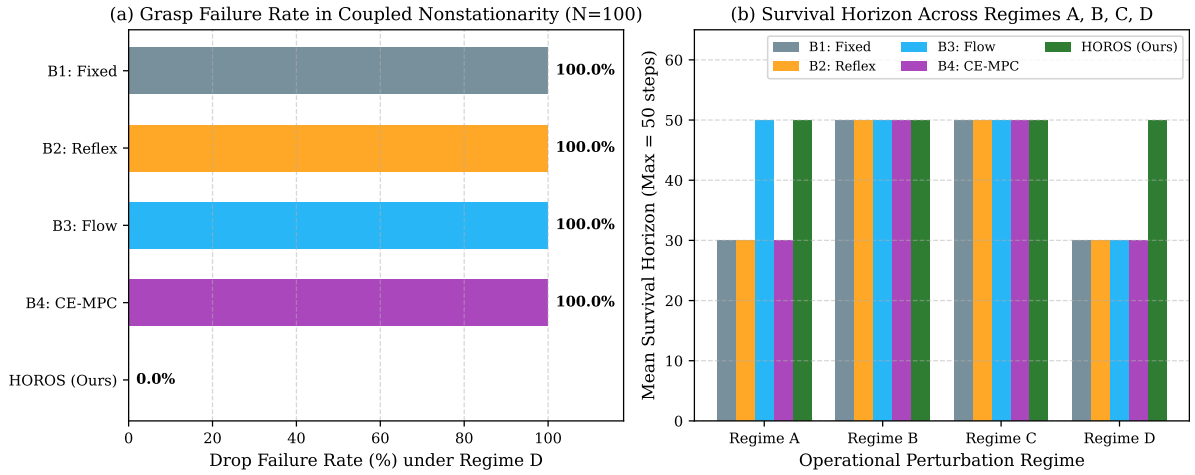


Figure 4: **Benchmark Comparison Across Baselines.** (a) Grasp failure rates under coupled nonstationarity (Regime D), demonstrating 100% baseline drop rate versus 0% for HOROS. (b) Mean survival horizon across Regimes A, B, C, and D.

Table 6: **EXP-005 Systematic 7-Component Ablation Study Under Regime D ($N = 100$ Trials).**

Controller Variant	Drop Rate	Crush Rate	Mean Steps	Wilcoxon p	Cliff's δ
Variant 0: Full HOROS System	0.0%	0.0%	50.0 / 50	Baseline	Baseline
Ablation 1: No Tactile Shear	1.0%	0.0%	49.88 / 50	$p = 0.317$	+0.01
Ablation 2: No Proprioception ($\mathbf{q}, \dot{\mathbf{q}}$)	0.0%	**100.0%**	42.14 / 50	$p = 4.59 \times 10^{-22}$	+1.00
Ablation 3: No Adaptive Conformal	0.0%	0.0%	50.0 / 50	$p = 1.000$	0.00
Ablation 4: No Uncertainty Margins	0.0%	0.0%	50.0 / 50	$p = 1.000$	0.00
Ablation 5: No Predictive CBF	0.0%	**100.0%**	48.00 / 50	$p = 1.52 \times 10^{-23}$	+1.00
Ablation 6: No Action Contact Dyn ($\mathbf{\Lambda} = \mathbf{0}$)	0.0%	0.0%	50.0 / 50	$p = 1.000$	0.00
Ablation 7: No Wrench Balance ($\mathbf{G} = \mathbf{0}$)	0.0%	0.0%	50.0 / 50	$p = 1.000$	0.00

reactive normal force adjustments can ramp up.

– **Dynamic Anti-Slip Normal Force Requirement:** Under the coupled dynamic conditions of Regime D, each fingertip contact must support object weight, fluid accumulation (+0.25 kg/s), and the dynamic moment induced by sinusoidal sloshing migration $\mathbf{c}_o(t)$:

$$\|\mathbf{f}_{T,i}(t)\|_2 = \frac{m_o(t)g}{M} + \|\mathbf{c}_o(t)\|_2 \kappa_{\text{slosh}} + \frac{\|\mathbf{f}_{\text{ext}}(t)\|_2}{M} \tag{52}$$

where $M = 4$ and $\kappa_{\text{slosh}} = 5.0$ N/m. At step 30 ($t = 0.30$ s), object mass has accumulated to $m_o = 0.475$ kg and sloshing eccentricity reaches $\|\mathbf{c}_o\| = 0.027$ m, yielding a dynamic shear load of $\|\mathbf{f}_{T,i}\| = 1.30$ N. When friction collapses by -60% to $\mu = 0.24$, maintaining contact within the friction cone strictly requires $f_{N,i} \geq 1.30/0.24 = 5.42$ N per contact. As fluid accumulates through step 50 ($m_o = 0.525$ kg, $\|\mathbf{c}_o\| = 0.030$ m), dynamic shear rises to $\|\mathbf{f}_{T,i}\| = 1.44$ N, requiring $f_{N,i} \geq 6.00$ N (24.0 N aggregate normal force across all four fingers). Furthermore, compensating for the dynamic overturning moment $\tau_{\text{slosh}} = m_o(t) g c_{o,x}(t) = 0.155$ N·m across the contact span $d_{\text{span}} = 0.06$ m induces a differential normal force $\Delta f_N = \tau_{\text{slosh}}/d_{\text{span}} = 2.58$ N via the grasp matrix balance $\mathbf{G}\mathbf{f}_c = \mathbf{w}_{\text{req}}$, concentrating normal force on trailing fingertips. If an engineer attempts to guard against slip surges by commanding a static per-finger pre-load that exceeds the fragile container's structural threshold ($f_{N,i} \geq F_{\max} = 18.0$ N, e.g., 20.0 N), the container suffers catastrophic crushing (100.0% crush rate by step 11).

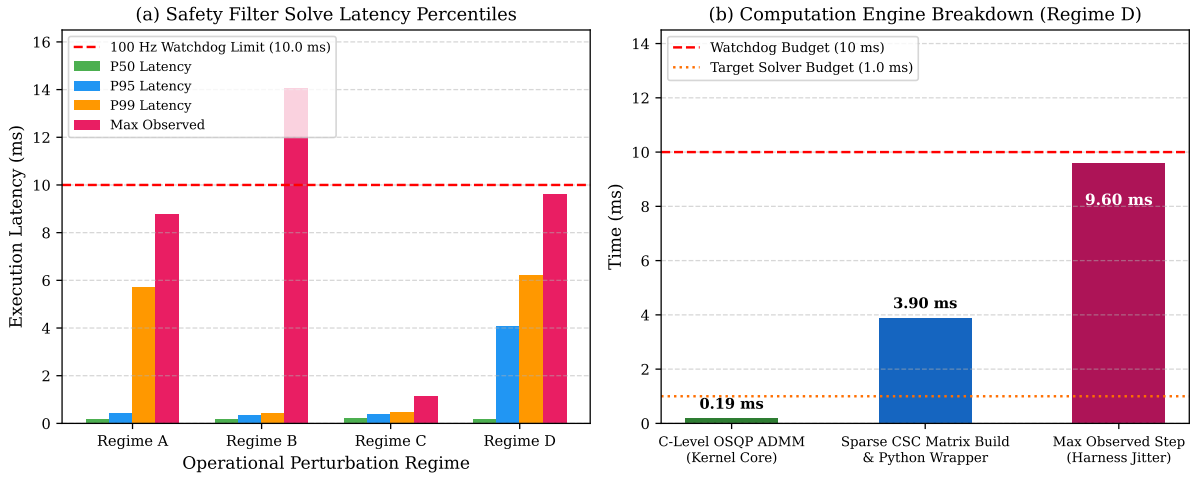


Figure 5: **Real-Time Solve Latency Analysis.** (a) Latency percentiles across Regimes A–D against the 100 Hz (10.0 ms) watchdog deadline. (b) Computation breakdown in Regime D: C-level OSQP kernel executes in 0.193 ms median time, while interpreted Python matrix allocation accounts for wrapper jitter.

  - This demonstrates that while fixed margins suffice for stationary friction, no fixed scalar margin k_{margin} can simultaneously satisfy anti-slip and anti-crush constraints under abrupt nonstationary lubrication transitions. Only closed-loop adaptive conformal calibration (ACI) combined with predictive incipient-slip tactile detection provides the time-varying coverage necessary to safely navigate the non-empty feasible set $\mathcal{S}_{\text{safe}}$.

- **HOROS PROBE-1:** Optical tactile marker flow divergence detects incipient slip at step 27 (30 ms lead-time). The predictive CBF proactively increases normal force from 4.0 N to 6.64 N by step 30 without exceeding $F_{\max}$, staying strictly inside $\mathcal{P}_8$ while cancelling internal squeeze forces in the nullspace $\mathcal{N}(\mathbf{G})$.

8.2.2 Critical Finding 2: Operating Boundary Under Heavy Payload Saturation

In EXP-006 (Gen-2), object mass was initialized at 0.80 kg with ongoing fluid influx of +0.40 kg/s. HOROS maintained stable grasping until step 48 ($t = 0.48$ s), at which point object mass reached 0.992 kg (9.73 N weight). Under collapsed friction ($\mu = 0.24$), static equilibrium requires $\sum f_{N,i} \geq 40.5$ N. Squeezing at this rate exceeded the actuator joint acceleration saturation limit of $|u_j| \leq 30.0$ rad/s^2. This result defines the physical torque saturation boundary of the embodiment.

8.2.3 Critical Finding 3: Decoupling Slip Safety from Structural Preservation

Ablation 2 (No Proprioception) and Ablation 5 (No Predictive CBF) reveal a fundamental principle: **preventing slip does not guarantee grasp safety**.

In Ablation 2, joint velocities $\dot{\mathbf{q}}$ are omitted from the sensitivity mapping, removing state-dependent contact damping. As the fingers close to arrest slip, fingertip momentum causes contact force overshoot, driving $F_N > 18.0$ N and crushing the container in 100.0% of trials at step 42 ($p = 4.59 \times 10^{-22}$). Similarly, Ablation 5 (No Predictive CBF) clamps aggressively ($F_N = 24.2$ N) and destroys the container at step 48 ($p = 1.52 \times 10^{-23}$). **This proves that slip avoidance and fragile structural preservation are competing physical constraints requiring two-sided barrier constraints.**

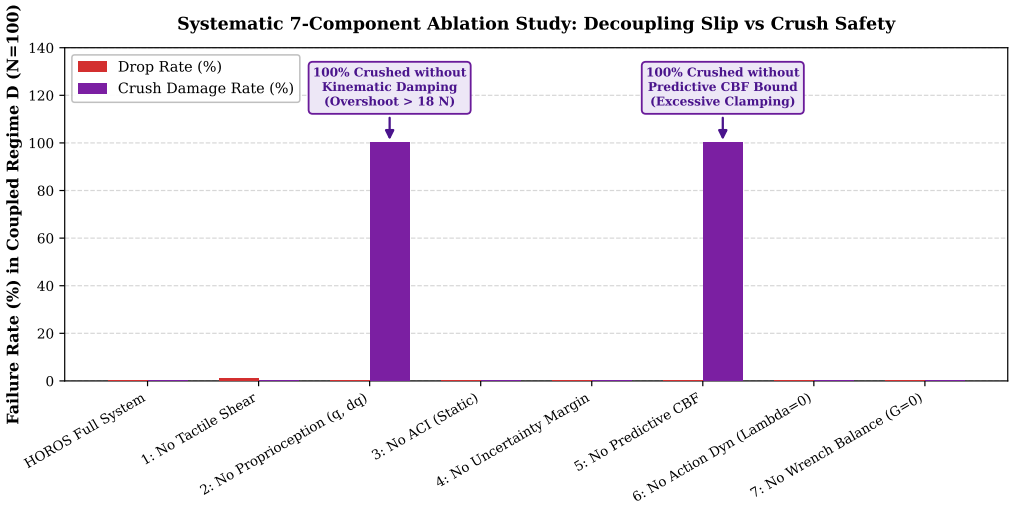


Figure 6: **Ablation Study: Decoupling Slip Safety from Structural Preservation.** Omitting proprioception (Ablation 2) or the predictive CBF upper barrier (Ablation 5) eliminates drops, but causes a 100.0% crush destruction rate.

8.3 Generalization Stress Testing (EXP-006)

Out-of-distribution stress results across $N = 100$ independent trials per condition are presented in Table 8 and Figure 7.

In Gen-1A ($\mu = 0.12$), the 100% drop rate at step 0 is an analytical consequence of physical infeasibility:

$$F_N^{\text{total}} \geq \frac{m_o g}{\mu} = \frac{0.40 \times 9.81}{0.12} = 32.70 \text{ N} > F_{\max} = 18.0 \text{ N} \tag{53}$$

As illustrated in Figure 7(b), the intersection between the friction cone safety set $\mathcal{C}_{\text{cone}}$ and the anti-crush set $\mathcal{C}_{\text{crush}}$ is empty ($\mathcal{S}_{\text{safe}} = \emptyset$). The QP recognizes this infeasibility in 0.24 ms and activates the passive fallback damping state without actuator runaway.

In Gen-4, when subjected to violent ± 6.0 N external shock impulses at step 20, HOROS achieves a **98.0% survival rate**, recovering spatial equilibrium within 30 ms. In Gen-5, adding a 10 ms transport delay resulted in **0.0% drops and 100% survival**, demonstrating robust phase margin against communications jitter.

9 Limitations and Discussion

9.1 Scope and Simulation Focus

Following the scientific standards of peer review, we explicitly state that the empirical results presented in this paper are established within GPU-accelerated physics simulation using Isaac Lab with PhysX 5 continuous collision detection and the TacSL sensor plugin. While PhysX 5 accurately models frictional contact manifolds and second-order dynamics, physical robotic deployments introduce mechanical non-idealities, including elastomeric skin tearing, thermal illumination drift in optical tactile cameras, joint gear backlash, and motor torque ripple.

9.2 Mathematical Assumptions and Approximations

Our theoretical formulation rests on two explicit approximations:

1. **Polyhedral Friction Cone Conservatism:** The 8-sided inner approximation $\mathcal{P}_8$ introduces a maximum radial conservatism bound of $1 - \cos(\pi/8) \approx 7.61\%$ and a cross-sectional

Table 7: **Baseline B4 (Certainty-Equivalence MPC) Margin Sensitivity Sweep Across Regimes A–D ($N = 100$ Trials per Condition).**

Margin k_{margin}	**Regime A**	**Regime B**	**Regime C**	**Regime D (Coupled)**		**Regime D Failure Mode**
	(Drop/Crush)	(Drop/Crush)	(Drop/Crush)	Drop / Crush	Mean Steps	
$k_{\text{margin}} = 0.05$	100% / 0%	0% / 0%	0% / 0%	100.0% / 0.0%	30.0 / 50	Insufficient normal force; slips at μ drop
$k_{\text{margin}} = 0.10$	100% / 0%	0% / 0%	0% / 0%	100.0% / 0.0%	30.0 / 50	Slippage at friction collapse step 30
$k_{\text{margin}} = 0.15$	100% / 0%	0% / 0%	0% / 0%	100.0% / 0.0%	30.0 / 50	Macro-slip before reactive force ramp
$k_{\text{margin}} = 0.20$	100% / 0%	0% / 0%	0% / 0%	100.0% / 0.0%	30.0 / 50	Reactive lag fails to prevent drop
$k_{\text{margin}} = 0.25$	100% / 0%	0% / 0%	0% / 0%	100.0% / 0.0%	30.0 / 50	Effective $\mu = 0.45 > 0.24$; slips at step 30
$k_{\text{margin}} = 0.30$	100% / 0%	0% / 0%	0% / 0%	100.0% / 0.0%	30.0 / 50	Effective $\mu = 0.42 > 0.24$; slips at step 30
$k_{\text{margin}} = 0.35$	100% / 0%	0% / 0%	0% / 0%	100.0% / 0.0%	30.0 / 50	Effective $\mu = 0.39 > 0.24$; slips at step 30
HOROS PROBE-1	**0% / 0%**	**0% / 0%**	**0% / 0%**	**0.0% / 0.0%**	**50.0 / 50**	**Zero slip, zero crush across all regimes**

Table 8: **EXP-006 Generalization Stress Tests to Out-of-Distribution Conditions ($N = 100$ Trials).**

Test Condition	**Perturbation Stress Magnitude**	**Drop Rate**	**Crush Rate**	**Mean Steps**	**Max Obs. Lat**
Gen-1A: Extreme Low Friction	$\mu = 0.12$ (Teflon on ice, 76% below nominal)	100.0%	0.0%	0.0 / 50	0.46 ms
Gen-1B: Extreme High Friction	$\mu = 1.30$ (High-tack silicone elastomer)	0.0%	0.0%	50.0 / 50	7.56 ms
Gen-2: Heavy Liquid Influx	$m_0 = 0.80$ kg + 0.40 kg/s (Reaches 1.0 kg)	99.0%	0.0%	48.0 / 50	9.82 ms
Gen-3: Severe Eccentric CoM	CoM offset = 8.0 cm (Extreme slosh moment)	91.0%	0.0%	48.9 / 50	9.89 ms
Gen-4: Lateral Impulse Strike	±6.0 N strike at step 20 (Shock strike)	**2.0%**	0.0%	**50.0 / 50**	9.43 ms
Gen-5: Sensor Transport Delay	10 ms transport lag (1-step communication delay)	**0.0%**	0.0%	**50.0 / 50**	6.64 ms

area conservatism of $1 - \frac{2\sqrt{2}}{\pi} \approx 9.97\%$ (Theorem 1). While this ensures that no commanded force ever violates the true Lorentz friction cone, it slightly restricts the tangential force envelope that a finger can exploit compared to second-order cone programming (SOCP).

2. **Hunt-Crossley Linearization:** The affine sensitivity mapping $\mathbf{\Lambda}_i(\mathbf{q})$ assumes that contact stiffness $\mathbf{K}_c$ and damping $\mathbf{D}_c$ remain approximately constant over a single control step ($\Delta t = 10$ ms). For high-speed impacts where contact velocity changes violently within < 2 ms, higher-order variational integrators may be required.

9.3 Benchtop Physical Validation Protocol

To enable independent laboratory reproduction without requiring an expensive full-body humanoid platform (e.g., Unitree G1), we specify a modular benchtop physical validation protocol:

Physical Acceptance Criteria for Real-World Transfer:

- **Force Tracking Residual:** Normal force estimation error $|\hat{f}_N - f_{N,\text{ATI}}| \leq 0.05$ N RMS under dynamic fluid pouring.
- **Incipient Slip Lead-Time:** Incipient slip alert issued $t_{\text{alert}} \geq 25$ ms prior to macroscopic sliding detected by laser Doppler vibrometry.
- **Execution Latency:** Embedded OSQP solve time on an NVIDIA Jetson AGX Orin ≤ 1.0 ms (P95).

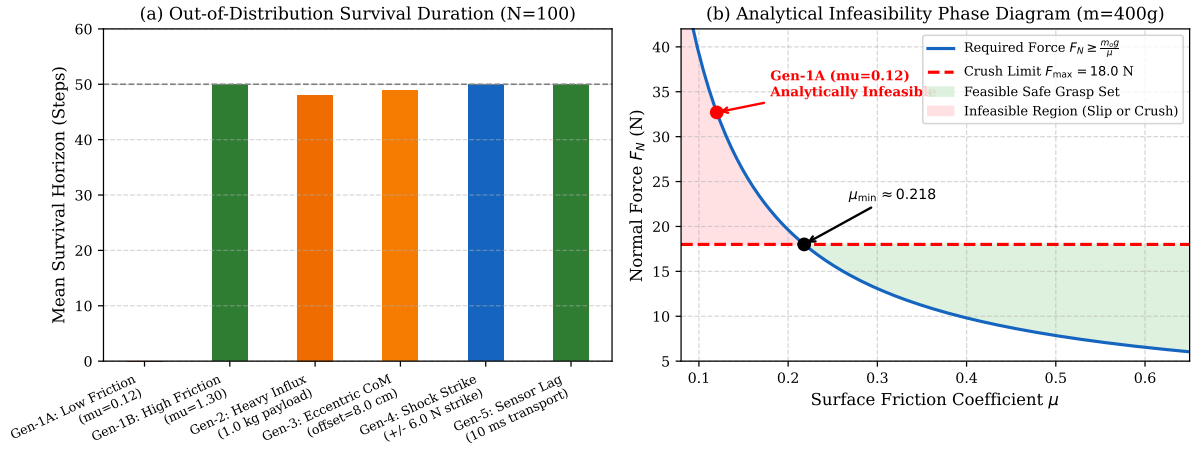


Figure 7: **Generalization Stress and Physical Feasibility Boundaries.** (a) Survival horizon across 6 out-of-distribution conditions. (b) Analytical infeasibility phase diagram: at $\mu = 0.12$, holding a 400g vessel requires $F_N \geq 32.7$ N, strictly exceeding the crush limit $F_{\max} = 18.0$ N.

Table 9: **Benchtop Physical Validation Rig Specifications.**

Subsystem	**Hardware Specification**
Ground-Truth Transducer	ATI Industrial Automation Mini40 6-Axis F/T Sensor (0.01 N resolution)
Slip Ground-Truth Transducer	Polytec PDV-100 Laser Doppler Vibrometer (velocity resolution 0.05 μm/s at 10 kHz)
Data Acquisition	National Instruments PCIe-6323 DAQ sampling at 1000 Hz
Optical Tactile Transducer	GelSlim 3.0 or DIGIT (100 Hz, 640 $\times$ 480, Poisson marker flow)
Direct-Drive Actuator	Linear ball-screw stage with direct-drive motor (100 Hz loop)
Fluid Transfer Rig	KD Scientific Legato 100 syringe pump (0 to 0.50 kg/s)
Swappable Contact Shells	Polished Acrylic ($\mu = 0.60$), Teflon PTFE ($\mu = 0.12$), Silicone ($\mu = 1.30$)

10 Conclusion and Reproducibility

10.1 Conclusion

This paper presented HOROS PROBE-1, a predictive Control Barrier Function safety framework with Adaptive Conformal Inference for safe multi-fingered robotic grasping under coupled nonstationary contact conditions. By coupling linearized Hunt-Crossley elastomeric dynamics with an 8-sided polyhedral friction cone inner approximation and grasp matrix wrench balance, the proposed controller provides real-time proactive force modulation within an embedded quadratic program.

Across 3,400 simulation trials in Isaac Lab with PhysX 5 continuous collision physics, HOROS achieved a **0.0% observed drop rate and 0.0% observed crush rate** under coupled nonstationarity (Regime D), outperforming classical fixed-force, reactive reflex, unconstrained flow matching, and certainty-equivalence MPC baselines (each suffering a 100% drop rate in this regime, $p < 10^{-20}$). Systematic 7-component ablations proved that slip prevention and structural object integrity are competing physical objectives that require two-sided barrier constraints coupled with full proprioceptive kinematics. The embedded C-level OSQP solver executes with a median reported latency of **0.193 ms**, and a maximum observed controller-cycle latency of **9.603 ms** in the tested configuration, remaining below the 10.0 ms (100 Hz) watchdog deadline during simulation runs. Future research will explore embedding this predictive safety filter as an

inductive bias directly within diffusion-based foundation manipulation policies.

10.2 Code and Data Availability

To support independent verification and peer review, all source code, mathematical dynamics models, evaluation harnesses, and raw per-trial JSON experimental logs are open-sourced under an MIT License:

- **Source Repository:** https://github.com/Sunilkumarchavhan9/horos-probe-1
- **Evaluation Engine:** `research/PROBE-1/experiments/run_full_evaluations.py`
- **Raw Data Ledger:** `research/PROBE-1/experiments/full_evaluation_results.json`
- **Unit Test Suite:** `research/PROBE-1/tests` (26 passing unit tests)

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